Text

Video Lecture 0: Overview of Linear Algebra

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Outline

- This video lecture gives an overview of the entire course (necessarily vaguely).
- You may want to return to it later to see how the "trees" fit together into a "forest"; don't worry if some things don't yet make sense.
- LinAlg sits at the crossroads of math, science, & engineering; it's ubiquitous in mathematics, and math-based disciplines.
- LinAlg is a beautiful subject, with a rich interplay of algebraic, geometrical, and analytic thinking.

Systems of Equations $\to$ Matrices

LinAlg's origins are in solving systems of linear equations, e.g., $\begin{cases} 2x + 3y = 4 \\ 4x + 7y = 6 \end{cases} \iff \begin{bmatrix} 2 & 3 \\ 4 & 7 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 4 \\ 6 \end{bmatrix}$, which we will rewrite as a single *vector-matrix* equation: $A\vec{x} = \vec{b}$.

A matrix A also represents a special kind of function $\mathbb{R}^n \to \mathbb{R}^m$, one which is *linear*.

Calculus studies linear approximations to more general functions (e.g., *the Jacobian matrix* of $\left[\frac{\partial_i f}{\partial x_j}\right]$).

Outside of calculus, many situations deal with *vectors* of values, which we want to understand: (e.g., digitized images, search engines, "big data", etc.). LinAlg can reveal patterns that are otherwise invisible, *weighting* vector components to clarify what's happening.

Flexible Thinking: Concrete $\leftrightarrow$ Abstract

- Use both concrete and abstract points of view to understand matrices as *linear transformations*, and homogeneous solutions as *vector subspaces*.
- *Key Idea:* Flexibility in choosing our point of view (coordinate system) simplifies formulae and computations. It's similar to switching to polar coordinates, but on steroids.
- We can often reduce the n^2 complexity of an $n \times n$ matrix to just n eigenvalues (or singular values) that tell us what we care about.
- Factoring a matrix into more easily understood pieces is a theme. The main ones are *Row Reduction* $A = LU$, *Diagonalization* $A = PDP^{-1}$, *Gram-Schmidt* $A = QR$, and *Singular Value Decomposition (SVD)* $A = U\Sigma V$.

Themes & Lay's Textbook

Video lectures are organized by theme to be textbook independent. Correspondence to Lay is indicated.

A. Applications: Pagerank, JPG compression, Principal Component Analysis (image processing), etc.

B. Bases & SuBspaces: $\mathrm{Col}\, A$, $\mathrm{Nul}\, A$, etc., dimension, rank, nullity; Lay § 4.1–7

D. Determinants: Theory, computations & geometric meaning. Lay § 3.1–3

E. Equations: Solving systems, row reduction, linear (in)dependence. Lay § 1.1–5, 1.7

F. Factorizations & Decompositions: LU, QR, Diagonalization PDP^{-1}, SVD $U\Sigma V$. Lay § 5.1–4, 7.1–4

G. Geometry: Scalar products, orthogonal & orthonormal bases, projections, Gram-Schmidt. Lay § 6.1–5

M. Matrices & linear transformations: Lay § 1.8–9, 2.1–35