Video Lecture M6: Matrix Operations 2

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Outline & Objectives

- Memorize the definition of the *transpose* A^T of a matrix A and compute examples.
- Generalize some algebraic properties for numbers to matrix multiplication, and demonstrate that others fail.
- Apply properties of these operations in examples and as part of reasoning in proofs.

Algebraic properties of matrix multiplication

Recall: $AB = [A\vec{b}_1 \ A\vec{b}_2 \ \cdots \ A\vec{b}_p]$ where $(m \times n)(n \times p) \mapsto (m \times p)$

Proposition

Let A, B, C be matrices of appropriate sizes, $r \in \mathbb{R}$:

1. $A(BC) = (AB)C$
2. $A(B + C) = AB + AC$
3. $(A + B)C = AC + BC$
4. $r(AB) = (rA)B = A(rB)$
5. $I_m A = A = A I_n$
6. *BUT* $AB \neq BA$ *OFTEN*

Think about dimensions for AB versus BA.

Can we make sense of A^4?
If $AB = 0$, then must $A = 0$ or $B = 0$?
If $AC = BC$ and $C \neq 0$ then must $A = B$?

Ex: Find examples of $A, B, C \in \mathbb{R}^{2\times 2}$ where (1) $AB \neq BA$,
(2) $AB = 0$, but neither A nor B is zero, and
(3) $AC = BC$, but $A \neq B$.

Matrix Transposes

Definition

For $m \times n$ matrix A, define A^T to be the $n \times m$ matrix whose rows are the columns of A, i.e., $A = (a_{ij}) \iff A^T = (a_{ji})$.

$$\begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \end{bmatrix}^T = \begin{bmatrix} 1 & 5 & 9 \\ 2 & 6 & 10 \\ 3 & 7 & 11 \\ 4 & 8 & 12 \end{bmatrix}, \begin{bmatrix} a & b & c \\ d & e & f \\ g & h & j \end{bmatrix}^T = \begin{bmatrix} a & d & g \\ b & e & h \\ c & f & j \end{bmatrix}.$$

Proposition

1. $(A^T)^T = A$
2. $(A+B)^T = A^T + B^T$
3. $(rA)^T = rA^T$
4. $(AB)^T = B^T A^T \neq A^T B^T$

Ex: Find an example of two 2×2 matrices A and B such that $(AB)^T \neq A^T B^T$.