Video Lecture M5: Matrix Operations

Tom Roby

Outline & Objectives

- Understand matrix addition and scalar multiplication conceptually and be able to compute examples.
- Know at least two different ways to multiply matrices and demonstrate why they are equivalent.
- Conceptualize matrix multiplication as composition of the corresponding transformations.

Matrix addition and scalar multiplication

Just think of them as "rectangular vectors": add corresponding terms or scale each term.

$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$, $B = \begin{bmatrix} 0 & -1 & 3 \\ 0 & -2 & 0 \end{bmatrix}$, and $C = \begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$.

When can we add two matrices?

When can we multiply a matrix by a scalar?

What matrix M satisfies $A + M = A$ for every $m \times n$ matrix A?

Proposition

For any same size matrices A, B, C and scalars $r, s \in \mathbb{R}$:

1. $A + B = B + A$
2. $(A + B) + C = A + (B + C)$
3. $A + 0 = A$
4. $r(A + B) = rA + rB$
5. $(r + s)A = rA + sA$
6. $r(sA) = (rs)A$

Matrix Multiplication

We want matrix multiplication to reflect composition of linear maps: $\mathbb{R}^p \xrightarrow{B} \mathbb{R}^n \xrightarrow{A} \mathbb{R}^m$ by $\vec{x} \mapsto B\vec{x} \mapsto A(B\vec{x}) = (AB)\vec{x}$.

Definition (matrix multiplication by columns)

If $B = \begin{bmatrix} \vec{b}_1 & \vec{b}_2 & \cdots & \vec{b}_p \end{bmatrix}$, *then* $AB = \begin{bmatrix} A\vec{b}_1 & A\vec{b}_2 & \cdots & A\vec{b}_p \end{bmatrix}$

Each column of AB is lin. comb. of cols of A w/ weights from corresponding column of B. $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{bmatrix}, B = \begin{bmatrix} -2 & 1 \\ 3 & 2 \end{bmatrix}$.

When can we multiply two matrices?

Proposition (Row-Column Rule for matrix multiplication)

$(AB)_{ij} = (i\text{th row of } A) \cdot (j\text{th column of } B) = a_{i1}b_{1j} + \cdots + a_{in}b_{nj}$.