Video Lecture M2: Linear Transformations

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Outline & Objectives

- Analyze matrix multiplication as representing a *transformation* (function) $T : \mathbb{R}^n \to \mathbb{R}^m$.
- Translate natural questions about a matrix transformation to solving SLE.

Matrix Transformations

Key Idea: Consider $A\vec{x} = \vec{y}$ as a *function ("transformation")*:
$T : \mathbb{R}^n \to \mathbb{R}^m$ via $\vec{y} = T(\vec{x}) = A\vec{x}$.

Let $A = \begin{bmatrix} 3 & 2 \\ 0 & 1 \\ 1 & 2 \end{bmatrix}$. What is $T(1,2)$? (evaluation)
Is $(1,2,3)$ in the range of T? (existence)
Does more than one $\vec{x} \in \mathbb{R}^n$ map to $(1,2,3)$? (uniqueness)

Same questions for $S : \mathbb{R}^3 \to \mathbb{R}^2$ by $S(\vec{x}) = B\vec{x}$, where
$B = \begin{bmatrix} 2 & 1 & -1 \\ 1 & 3 & 0 \end{bmatrix}$, $\vec{x} = (2,1,3)$, and $\vec{y} = (2,4)$.

Examples from $\mathbb{R}^2$ to $\mathbb{R}^2$

Simple examples are a good place to start. I highly recommend the applet:

http://math.mercyhurst.edu/~lwilliams/Applets/LinearTransformations.html

Consider the matrix transformations given by the following:

1. $A = \begin{bmatrix} 3 & 0 \\ 0 & \frac{1}{2} \end{bmatrix}$
2. $B = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$
3. $C = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$.
4. $D = \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}$