Video Lecture F2: Geometry of Eigenvectors

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Outline & Objectives

- Prove that the set of all eigenvectors corresponding to an eigenvalue λ forms a vector space, called the *eigenspace of λ*, sometimes denoted E_λ.
- Analyze the action of simple matrices in $\mathbb{R}^2$ and $\mathbb{R}^3$ to better grasp the "geometry" of eigenspaces.
- Prove that eigenvectors corresponding to *distinct* eigenvalues are linearly independent.

Actions on Eigenspaces

Proposition (Eigenvectors for same λ form subspace)

If $\vec{v}$ and $\vec{w}$ are eigenvectors for λ, then so is $c\vec{v} + d\vec{w}$ for any c, d in $\mathbb{R}$; hence, $E_\lambda := \{\text{all eigenvectors for } \lambda\}$ is a subspace of $\mathbb{R}^n$.

Proof: $A(c\vec{v} + d\vec{w}) = cA\vec{v} + dA\vec{w} = c\lambda\vec{v} + d\lambda\vec{w} = \lambda(c\vec{v} + d\vec{w})$.

$$\begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}.$$

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & -5 \end{bmatrix}.$$

$$\begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & -5 \end{bmatrix}.$$

$$\begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & -5 \end{bmatrix}.$$

Eigenvectors for distinct eigenvalues

Theorem (Eigenvectors for distinct eigenvalues are lin indep)

Let $\vec{v}_1, \vec{v}_2, \ldots \vec{v}_r$ be eigenvectors corresponding to distinct eigenvalues $\lambda_1, \lambda_2, \ldots, \lambda_r$ for $A \in \mathbb{R}^{n\times n}$. Then $\{\vec{v}_1, \vec{v}_2, \ldots \vec{v}_r\}$ is linearly independent.