Video Lecture E9: Solving homogenous systems

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Outline & Objectives

- Compute nontrivial solutions to a homomegenous system $A\vec{x} = \vec{0}$.

Homogenous systems

Definition

A system of linear equations whose matrix form is $A\vec{x} = \vec{0}$ is called homogenous. (Here A is $m \times n$). A solution $\vec{x} \in \mathbb{R}^n$ is called trivial if $\vec{x} = \vec{0}$ and nontrivial if $\vec{x} \neq \vec{0}$.

$$\left[\begin{array}{ccc|c} -1 & 5 & 2 & b_1 \\ 3 & -14 & -7 & b_2 \\ -2 & 10 & 4 & b_3 \end{array}\right] \to \left[\begin{array}{ccc|c} 1 & -5 & -2 & -b_1 \\ 0 & 1 & -1 & 3b_1 + b_2 \\ 0 & 0 & 0 & -2b_1 + b_3 \end{array}\right]$$

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$x_1 = 7x_3$

$x_2 = x_3$

x_3 is free

So any vector of the form $\begin{bmatrix} 7k \\ k \\ k \end{bmatrix} = k \begin{bmatrix} 7 \\ 1 \\ 1 \end{bmatrix}$ is a solution (line in $\mathbb{R}^3$).

Corollary

The homogenous eqn. $A\vec{x} = \vec{0}$ has a nontrivial solution $\iff$ there is at least one free variable.

Another example

$$\begin{cases} x_1 \phantom{{}-x_2} + x_3 + x_4 = 0 \\ 2x_1 - x_2 \phantom{{}+3x_3} + x_4 = 0 \\ x_1 + x_2 + 3x_3 + 2x_4 = 0 \end{cases} \mapsto \begin{bmatrix} 1 & 0 & 1 & 1 \\ 2 & -1 & 0 & 1 \\ 1 & 1 & 3 & 2 \end{bmatrix} \mapsto \begin{bmatrix} 1 & 0 & 1 & 1 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

True or False?

T/F: If a matrix A has RREF U with no row all zeroes, then $A\vec{x} = \vec{0}$ has no solutions.

T/F: If a matrix A has RREF U with no row all zeroes, then $A\vec{x} = \vec{0}$ has only the trivial solution.

T/F: If a square matrix A has a pivot in each row, then $A\vec{x} = \vec{0}$ has only the trivial solution.