Video Lecture E4: Reduced Echelon Form

Tom Roby

Outline & Objectives

- Systematically use row operations to compute a matrix to its *unique* **reduced (row) echelon form (RREF)**.
- Analyze the solution set of systems of linear equations via their RREF.

Definition

A matrix A is in reduced (row) echelon form (RREF) if it satisfies:

1. *All nonzero rows are above any rows of all zeros.*
2. *The leading (first $\neq 0$) entry in each nonzero row is a 1.*
3. *Each leading 1 is the only nonzero entry in its column.*
4. *The leading 1 in each row is in a column to the right of the leading 1s above it.*

Pivots

Definition

A pivot position of A a location of a leading 1 in RREF of A.
A pivot column is one that contains a pivot position.

EG: $\left[\begin{array}{ccc|c} 1 & 3 & 4 & 7 \\ 0 & 3 & 6 & 3 \\ -1 & -3 & -2 & -9 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & -1 \end{array}\right]$,

EG: $\left[\begin{array}{ccc|c} -2 & -4 & -1 & -21 \\ -4 & 8 & 1 & 39 \\ 5 & 10 & 2 & 51 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 2 & 0 & 9 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 \end{array}\right]$,

EG: $\left[\begin{array}{ccc|c} 1 & -1 & -1 & -1 \\ 3 & 1 & 9 & 1 \\ 2 & 0 & 4 & 4 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 0 & 2 & 0 \\ 0 & 1 & 3 & 1 \\ 0 & 0 & 0 & 1 \end{array}\right]$

Parametric descriptions of solution sets

Definition

The variables corresponding to pivot columns in the matrix are called basic variables. Any other variables are called free variables.

EG: $\left[\begin{array}{ccc|c} 1 & 3 & 4 & 7 \\ 0 & 3 & 6 & 3 \\ -1 & -3 & -2 & -9 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 1 & -1 \end{array}\right]$,

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EG: $\left[\begin{array}{ccc|c} 1 & -1 & -1 & -1 \\ 3 & 1 & 9 & 1 \\ 2 & 0 & 4 & 4 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 0 & 2 & 0 \\ 0 & 1 & 3 & 1 \\ 0 & 0 & 0 & 1 \end{array}\right]$

Example

EG: Write the parametric description of the solution set.

$$\begin{cases} y + 2z = 3 \\ 2x - 6z = -8 \\ 3x + 6y + 3z = 6 \end{cases}$$