Video Lecture E2: Solving Systems of Linear Equations

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Outline & Objectives

- Linear systems are *equivalent* if they have the *same* solution set.
- *Row operations* change linear systems into equivalent linear systems.
- Using successive row operations, we change a linear system to a simpler one to find the solution.
- We can view this same process as working on corresponding *matrices* that represent the linear system.

Equivalent Systems of Linear Equations

Definition

We call two systems of linear equations equivalent if they have exactly the same solution set.

$$\begin{cases} 2x + 3y = 4 \\ 4x + 7y = 6 \end{cases} \quad \begin{cases} 2x + 3y = 4 \\ \qquad\ \ y = -2 \end{cases} \quad \begin{cases} 2x \qquad = 10 \\ \qquad\ y = -2 \end{cases}$$

Elementary Row Operations

Definition

The three elementary row operations are

1. *Replacement: Add a multiple of one row to another row;*
2. *Interchange: Interchange two rows; and*
3. *Scaling: Multiply one row by a nonzero constant.*

None of these operations changes the solution set of the SLE!

We call two linear systems *row-equivalent* if there is a sequence of row operations that takes one to the other.

So row-equivalent implies equivalent.

Solving a SLE with row operations

EG: Solve $\begin{cases} y + 2z = 3 \\ 2x - 6z = -8 \\ 3x + 6y - 2z = -4 \end{cases}$

DID YOU CHECK? SLE consistent? Solution unique?

What did we do?