Video Lecture D5: Determinants & Volume

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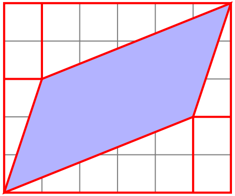
Outline & Objectives

- Reconcepualize $\det A$ as computing the (signed) volume of the parallelopiped in $\mathbb{R}^n$ defined by the column vectors of A.
- Understand $|\det A|$ as being the factor by which the linear transformation $T(\vec{x}) = A\vec{x}$ rescales the volume of any $S \subseteq \mathbb{R}^n$.

Areas of Parallelograms

Find area of the parallelogram det'd by the vectors $\begin{bmatrix} a \\ c \end{bmatrix}$ and $\begin{bmatrix} b \\ d \end{bmatrix}$.



Theorem

For $A \in \mathbb{R}^{n\times n}$, the volume of the parallelopiped determined by the columns of A is $|\det A|$.

Idea: WTS: Row/Col ops affect area and $|\det A|$ in the same way. This reduces to case where where A is diagonal, where thm is clear.

Linear Transformations

Theorem

Let $T : \mathbb{R}^n \to \mathbb{R}^n$ *have standard matrix* A*, i.e.,* $T(\vec{x}) = A\vec{x}$*. Let* $S \subseteq \mathbb{R}^n$ *be any region with finite volume. Then* $\mathrm{vol}\, T(S) = |\det A|\, \mathrm{vol}(S)$.