Video Lecture B2: Vector Subspaces

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Outline & Objectives

- Define *vector subspaces* and analyze whether given subspaces are subspaces or not.
- Demonstrate that the span of any set of vectors in V is a subspace of V.

Vector Subspaces

We often consider vector spaces that live inside ambient ones.

Definition

Call H a subspace of a vector space V if it satisfies:

a. $\vec{0} \in H$ *(Axiom 4)*

b. $\vec{u}, \vec{v} \in H \implies \vec{u} + \vec{v} \in H$ *(Axiom 1)*

c. $\vec{u} \in H$ *and* $c \in \mathbb{R} \implies c\vec{u} \in H$ *(Axiom 6)*

EG: $\{\vec{0}\}$ inside any V.

EG: $\left\{ \begin{bmatrix} a \\ 0 \\ b \end{bmatrix} : a, b \in \mathbb{R} \right\}$ in $\mathbb{R}^3$.

EG: $\mathbb{P}_3$ inside ??

Which are subspaces?

EG 1: $\left\{ \begin{bmatrix} a \\ a \end{bmatrix} : a \in \mathbb{R} \right\}$ in $\mathbb{R}^2$.

EG 2: $\{f : \mathbb{R} \to \mathbb{R} : f \text{ is differentiable}\}$ in ...

EG 3: $\left\{ \begin{bmatrix} a & a+b \\ 0 & b \end{bmatrix} : a, b \in \mathbb{R} \right\}$ in ...

EG 4: $\left\{ \begin{bmatrix} a \\ a^2 \end{bmatrix} : a \in \mathbb{R} \right\}$ in $\mathbb{R}^2$.

EG 5: $\{\vec{p}(t) = at^2 + at + a : a \in \mathbb{R}\}$ in $\mathbb{P}_2$.

EG 6: $\{\vec{p}(t) = t^2 + at + b : a, b \in \mathbb{R}\}$ in $\mathbb{P}_2$.

EG 7: $\left\{ \begin{bmatrix} a & 1 \\ 0 & b \end{bmatrix} : a, b \in \mathbb{R} \right\}$ in ...

Spans of sets of vectors

Theorem

Let $\vec{v}_1, \ldots, \vec{v}_p \in V$, some vector space. Then $\mathrm{Span}\{\vec{v}_1, \ldots, \vec{v}_p\}$ is a subspace of V.

Proof: Recall that $H := \mathrm{Span}\{\vec{v}_1, \ldots, \vec{v}_p\}$ is just $\{c_1\vec{v}_1 + \cdots + c_p\vec{v}_p : c_i \in \mathbb{R}\}$.

Why is $\vec{0} \in H$?

Why is H closed under vector addition?

Why is H closed under scalar multiplication?