Video Lecture B1: Vector Spaces

Tom Roby

Outline & Objectives

- Isolate the essential algebraic axioms we need for linear algebra, generalizing our notion of *vector* to include a much wider range of examples.
- Analyze whether certain sets satisfy the axioms necessary to be considered *vector spaces*.

What could we call a "vector"?

Q: What makes something a vector?

We want to add two of them: $\vec{v}, \vec{w} \to \vec{v} + \vec{w}$

We want to rescale them: $\vec{v}, k \to k\vec{v}$.

EG: Polynomials

EG: Doubly-infinite sequences

EG: Real-valued functions on R (or on $[a, b]$).

EG: Matrices in $\mathbb{R}^{m \times n}$.

Vector Space Axioms

Definition

A (real) vector space V is a set with two operations (addition and rescaling) which satisfy:

1. $\vec{u}, \vec{v} \in V \implies \vec{u} + \vec{v} \in V$.
2. $(\vec{u} + \vec{v}) + \vec{w} = \vec{u} + (\vec{v} + \vec{w})$
3. $\vec{u} + \vec{v} = \vec{v} + \vec{u}$.
4. $\exists \vec{0} \in V$ *s.t.* $\forall \vec{u} \in V,\ \vec{u} + \vec{0} = \vec{u}$.
5. $\forall \vec{u} \in V, \exists (-\vec{u}) \in V$ *s.t.* $(-\vec{u}) + \vec{u} = \vec{0}$.
6. $c \in \mathbb{R}$ *and* $\vec{u} \in V \implies c\vec{u} \in V$.
7. $c(\vec{u} + \vec{v}) = c\vec{u} + c\vec{v}$.
8. $(c + d)\vec{u} = c\vec{u} + d\vec{u}$.
9. $c(d\vec{u}) = (cd)\vec{u}$.
10. $1\vec{u} = \vec{u}$.

Proposition

$\forall c \in \mathbb{R}$ *and* $\vec{u} \in V$: *(1)* $0\vec{u} = \vec{0}$; *(2)* $c\vec{0} = \vec{0}$; *(3)* $(-1)\vec{u} = -\vec{u}$.