\( \Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-y^2/2} dy = P(Z < x) \) where \( Z \) is the standard normal random variable.

(1) Suppose we roll a die once, and let \( D \) be the value that is showing. Define \( X = (D - 3)^2 \). Find the p.m.f., \( \mathbb{E} \), Var of \( X \).

(2) Suppose we roll a die 9 times. Find the the best Poisson approximation for the probability to have one or two outcomes divisible by 3.

(3) An insurance company insures a large number of homes. The insured value, \( X \), of a randomly selected home is assumed to follow a distribution with density function

\[
f(x) = \begin{cases} \frac{c}{x^4} & x > 2, \\ 0 & \text{otherwise}. \end{cases}
\]

Find \( c \). After that find the c.d.f., p.d.f., \( \mathbb{E} \), Var of \( X \).

(4) Suppose \( X = \mathcal{N}(\mu, \sigma^2) \), \( P(X < 1) = 0.00135 = \Phi(-3) \) and \( P(X < 5) = 0.99865 = \Phi(3) \). Find \( \mu \) and \( \sigma \).

(5) Suppose we roll a die 18 times. Find the formula for the best possible normal approximation for the probability to have either 5 or 6 outcomes divisible by 3. Your answer should include \( \Phi \) twice.

(6) Let \( X = \text{Exp}(\lambda = 2) \) and \( Y = X^2 \). Find the formulas for c.d.f., p.d.f., \( \mathbb{E} \), Var of \( Y \). You are asked to write all derivatives and integrals explicitly, but do not have simplify or evaluate them.