MATH 3160 - Probability - Fall 2017
Test 2 sample problems

$\Phi(x) = \frac{1}{\sqrt{2\pi}} \int\limits_{-\infty}^{x} e^{-y^2/2} dy = \mathbb{P}(Z < x)$ where Z is the standard normal random variable.

(1) Suppose we roll a die once, and let D be the value that is showing. Define $X = (D-3)^2$. Find the p.m.f., $\mathbb{E}$, **Var** of X.

(2) Suppose we roll a die 9 times. Find the the best Poisson approximation for the probability to have one or two outcomes divisible by 3.

(3) An insurance company insures a large number of homes. The insured value, X, of a randomly selected home is assumed to follow a distribution with density function

$$f(x) = \begin{cases} \frac{c}{x^4} & x > 2, \\ 0 & \text{otherwise}. \end{cases}$$

Find c. After that find the c.d.f., p.d.f., $\mathbb{E}$, **Var** of X.

(4) Suppose $X = \mathcal{N}(\mu, \sigma^2)$, $P(X < 1) = 0.00135 = \Phi(-3)$ and $P(X < 5) = 0.99865 = \Phi(3)$. Find μ and σ.

(5) Suppose we roll a die 18 times. Find the formula for the best possible normal approximation for the probability to have either 5 or 6 outcomes divisible by 3. Your answer should include Φ twice.

(6) Let $X = \text{Exp}(\lambda = 2)$ and $Y = X^2$. Find the formulas for c.d.f., p.d.f., $\mathbb{E}$, **Var** of Y. You are asked to write all derivatives and integrals explicitly, but do not have simplify or evaluate them.