MATH 3160 - Probability - Fall 2017
Test 2 sample problems

Use the notation $\Phi(x)$ for the $\mathcal{N}(0,1)$ distribution function, that is
$\Phi(x) = \frac{1}{\sqrt{2\pi}} \int\limits_{-\infty}^{x} e^{-y^2/2} dy = \mathbb{P}(Z < x)$ where Z is the standard normal random variable.

. . . .

(1) Suppose we roll a die once, and let D be the value that is showing. Define $X = (D-3)^2$. Find the p.m.f., $\mathbb{E}$, **Var** of X.

(2) Suppose we roll a die 9 times. Find the the best Poisson approximation for the probability to have one or two outcomes divisible by 3.

(3) An insurance company insures a large number of homes. The insured value, X, of a randomly selected home is assumed to follow a distribution with density function

$$f(x) = \begin{cases} \frac{c}{x^4} & x > 2, \\ 0 & \text{otherwise}. \end{cases}$$

Find c. After that find the c.d.f., p.d.f., $\mathbb{E}$, **Var** of X.

(4) Suppose $X = \mathcal{N}(\mu, \sigma^2)$, $P(X < 1) = 0.00135 = \Phi(-3)$ and $P(X < 5) = 0.99865 = \Phi(3)$. Find μ and σ.

(5) Suppose we roll a die 18 times. Find the formula for the best possible normal approximation for the probability to have either 5 or 6 outcomes divisible by 3. Your answer should include Φ twice.

(6) Let $X = \text{Exp}(\lambda = 2)$ and $Y = X^2$. Find the formulas for c.d.f., p.d.f., $\mathbb{E}$, **Var** of Y. You are asked to write all derivatives and integrals explicitly, but do not have simplify or evaluate them.

Answers:

(1) $P(X=0) = P(X=9) = \frac{1}{6}$, $P(X=1) = P(X=4) = \frac{1}{3}$,
$\mathbb{E}X = \frac{9}{6} + \frac{1+4}{3} = \frac{19}{6}$,
$Var(X) = \frac{81}{6} + \frac{1+16}{3} - (\frac{19}{6})^2$. This is $\frac{329}{36}$, but on a test you do not need to simplify.

(2) $e^{-3}(3 + \frac{9}{2})$

(3) $c = 24$
$F_X(x) = 1 - \frac{8}{x^3}$ when $x > 2$ and 0 otherwise.
$f_X(x) = \frac{24}{x^4}$ when $x > 2$ and 0 otherwise.
$\mathbb{E}X = 3$, $\mathbb{E}X^2 = 12$, $\text{Var}(X) = 12 - 9 = 3$

(4) $\mu = 3$, $\sigma = \frac{2}{3}$.

(5) $\text{Bin}(n = 18, p = \frac{1}{3})$, $\mu = np = 6$, $\sigma^2 = np(1-p) = 4$, $X = 6 + 2Z$, $\mathbb{P}(4.5 < X < 6.5) = \mathbb{P}(4.5 < 6 + 2Z < 6.5) = \mathbb{P}(-0.75 < Z < 0.25) = \Phi(0.25) - \Phi(-0.75)$
This is $0.59871 - 0.22663 = 0.37208$ (you may or may not be asked to use the table on a test).

(6) Let $y = x^2$, $x = \sqrt{y}$. Then $F_Y(y) = \mathbb{P}(Y < y) = \mathbb{P}(X < x) = F_X(\sqrt{y}) = 1 - e^{-2\sqrt{y}}$
$f_Y(y) = e^{-2\sqrt{y}}/\sqrt{y}$
$\mathbb{E}Y = \mathbb{E}X^2 = \int_0^\infty 2x^2 e^{-2x} dx$
$\mathbb{E}Y^2 = \mathbb{E}X^4 = \int_0^\infty 2x^4 e^{-2x} dx$
$\text{Var}(Y) = \mathbb{E}Y^2 - (\mathbb{E}Y)^2$.
In fact $\text{Var}(Y) = 1.5 - (0.5)^2 = 1.25$, but this is not on the test.