MATH 3160 - Probability - Fall 2017
Test 1, Wednesday October 4, Answers

(1) Three balls are randomly withdrawn without replacement from a bowl containing **4** white and **3** black balls. What is the probability that one ball is white and the other two are black?

Solution and answer:

There are two solutions, by combinations and the multiplication rule, which briefly can be written as follows: $\dfrac{\binom{4}{1}\cdot\binom{3}{2}}{\binom{7}{3}} = \mathbf{3}\cdot\dfrac{\mathbf{4\cdot 3\cdot 2}}{\mathbf{7\cdot 6\cdot 5}} = \dfrac{\mathbf{12}}{\mathbf{35}}$

(2) There are **6** Large Blue balls, **3** Small Blue balls, and **2** Large Green balls. How many Small Green balls must be present in the box if size and color are independent when choosing a ball selected at random?

Solution and answer:

There are several solutions. We have $\mathbb{P}(\boldsymbol{S}) = \dfrac{\boldsymbol{3+x}}{\boldsymbol{11+x}}$, $\mathbb{P}(\boldsymbol{G}) = \dfrac{\boldsymbol{2+x}}{\boldsymbol{11+x}}$ $\mathbb{P}(\boldsymbol{S}\cap\boldsymbol{G}) = \dfrac{\boldsymbol{x}}{\boldsymbol{11+x}}$. Therefore we can solve $\dfrac{\boldsymbol{3+x}}{\boldsymbol{11+x}}\cdot\dfrac{\boldsymbol{2+x}}{\boldsymbol{11+x}} = \dfrac{\boldsymbol{x}}{\boldsymbol{11+x}}$ which is the same as $\boldsymbol{(3+x)(2+x) = x(11+x)}$ or $\boldsymbol{6+5x+x^2 = 11x+x^2}$ or $\boldsymbol{6 = 6x}$. Then $\boldsymbol{x = 1}$ is the answer.

One of the easiest is the following. If size and color are independent, then $\mathbb{P}(\boldsymbol{L}|\boldsymbol{B}) = \mathbb{P}(\boldsymbol{L})$ which means $\dfrac{\mathbf{6}}{\mathbf{3+6}} = \dfrac{\mathbf{8}}{\boldsymbol{11+x}}$ which implies $\boldsymbol{x = 1}$.

Even easier solution can be obtained from the multiplication rule $\dfrac{\mathbb{P}(\boldsymbol{G})}{\mathbb{P}(\boldsymbol{B})} = \dfrac{\mathbb{P}(\boldsymbol{G}|\boldsymbol{L})}{\mathbb{P}(\boldsymbol{B}|\boldsymbol{L})} = \dfrac{\mathbb{P}(\boldsymbol{G}|\boldsymbol{S})}{\mathbb{P}(\boldsymbol{B}|\boldsymbol{S})} = \dfrac{\mathbf{2}}{\mathbf{6}} = \dfrac{\boldsymbol{x}}{\mathbf{3}}$, and so $\boldsymbol{x = 1}$. Here is the contingency table:

	L	S	total
G	2	x	2+x
B	6	3	9
total	8	x+3	11+x

(3) Suppose that there are **14** boxes. Each box can contain pencils, markers, or both pencils and markers, or neither. We know that **5** boxes are empty, **5** boxes contain pencils, and **6** boxes contain markers.

How many boxes contain both pencils and markers?

Solution and answer:

2 because there are $\mathbf{14-5=9}$ non-empty boxes, and $\mathbf{6+5-9=2}$ boxes that have both pencils and markers

If a box is chosen at random, and we know that it contains a pencil, then what is the probability that it also contains a marker?

Solution and answer:

$\mathbb{P}(\boldsymbol{M}|\boldsymbol{P}) = \dfrac{\mathbb{P}(\boldsymbol{M}\cap\boldsymbol{P})}{\mathbb{P}(\boldsymbol{P})} = \dfrac{\mathbf{2}}{\mathbf{5}}$ which is the number of boxes that have both pencils and markers divided by the number of boxes with pencil

(4) Suppose that

- a flu test (correctly) indicates the presence of the flu $\frac{4}{5}$ of the times when the patient actually has the flu *(this is called the true positive rate)*;
- the same test (incorrectly) indicates the presence of flu $\frac{1}{4}$ of the times when flu is not actually present *(this is called the false positive rate)*;
- currently $\frac{1}{3}$ is the rate of the flu in the population.

For a random person, what is the probability that the flu test is positive?

Solution and answer:

$$\boxed{\mathbb{P}(test+) = \frac{4}{5}\cdot\frac{1}{3}+\frac{1}{4}\cdot\frac{2}{3} = \frac{4}{5\cdot 3}+\frac{1}{2\cdot 3} = \frac{8+5}{2\cdot 5\cdot 3} = \frac{13}{30}}$$

What is the probability that a random person actually has the flu, given that the flu test is positive?

Solution and answer:

$$\boxed{\mathbb{P}(flu|test+) = \frac{\mathbb{P}(flu\cap test+)}{\mathbb{P}(test+)} = \frac{\frac{4}{15}}{\frac{13}{30}} = \frac{8}{13}}$$

(5) There are **18** cards that are divided in three groups of cards, containing **5**, **6**, **7** cards respectively. Each group contains **2** red cards, and the other cards are black. We shuffle all these cards and select a random card, which happens to be black. What is the probability that this particular card was from the first group?

Solution and answer:

the first group contains **3** back cards, and the total number of black cars is $18-6=12$. Therefore the answer is $\frac{3}{12}=\frac{1}{4}$

Another solution is $\mathbb{P}(first|black) = \frac{\mathbb{P}(first\cap black)}{\mathbb{P}(black)} = \frac{\frac{3}{18}}{\frac{12}{18}} = \frac{1}{4}$