(1) Three balls are randomly withdrawn without replacement from a bowl containing 4 white and 3 black balls. What is the probability that one ball is white and the other two are black?

**Solution and answer:**

There are two solutions, by combinations and the multiplication rule, which briefly can be written as follows:

\[
\frac{\binom{4}{1} \cdot \binom{3}{2}}{\binom{7}{3}} = 3 \cdot \frac{4 \cdot 3 \cdot 2}{7 \cdot 6 \cdot 5} = \frac{12}{35}
\]

(2) There are 6 Large Blue balls, 3 Small Blue balls, and 2 Large Green balls. How many Small Green balls must be present in the box if size and color are independent when choosing a ball selected at random?

**Solution and answer:**

There are several solutions. We have

\[
P(S) = \frac{3 + x}{11 + x}, \quad P(G) = \frac{2 + x}{11 + x}, \quad P(S \cap G) = \frac{x}{11 + x}.
\]

Therefore we can solve

\[
\frac{3 + x}{11 + x} \cdot \frac{2 + x}{11 + x} = \frac{x}{11 + x}
\]

which is the same as \((3+x)(2+x) = x(11+x)\) or \(6 + 5x + x^2 = 11x + x^2\) or \(6 = 6x\). Then \(x = 1\) is the answer.

One of the easiest is the following. If size and color are independent, then \(P(L|B) = P(L)\) which means \(\frac{6}{3 + 6} = \frac{8}{11 + x}\) which implies \(x = 1\).

Even easier solution can be obtained from the multiplication rule

\[
P(G) = P(G|L) \cdot P(L) = \frac{P(G|S)}{P(B|S)} = \frac{2}{6} \cdot \frac{x}{3}, \text{ and so } x = 1.
\]

Here is the contingency table:

<table>
<thead>
<tr>
<th></th>
<th>L</th>
<th>S</th>
<th>total</th>
</tr>
</thead>
<tbody>
<tr>
<td>G</td>
<td>2</td>
<td>x</td>
<td>2+x</td>
</tr>
<tr>
<td>B</td>
<td>6</td>
<td>3</td>
<td>9</td>
</tr>
<tr>
<td>total</td>
<td>8</td>
<td>x+3</td>
<td>11+x</td>
</tr>
</tbody>
</table>

(3) Suppose that there are 14 boxes. Each box can contain pencils, markers, or both pencils and markers, or neither. We know that 5 boxes are empty, 5 boxes contain pencils, and 6 boxes contain markers.

How many boxes contain both pencils and markers?

**Solution and answer:**

2 because there are \(14 - 5 = 9\) non-empty boxes, and \(6 + 5 - 9 = 2\) boxes that have both pencils and markers.

If a box is chosen at random, and we know that it contains a pencil, then what is the probability that it also contains a marker?

**Solution and answer:**

\[
P(M|P) = \frac{P(M \cap P)}{P(P)} = \frac{2}{5}
\]

which is the number of boxes that have both pencils and markers divided by the number of boxes with pencil.
(4) Suppose that
- a flu test (correctly) indicates the presence of the flu \( \frac{4}{5} \) of the times when the patient actually has the flu (this is called the true positive rate);
- the same test (incorrectly) indicates the presence of flu \( \frac{1}{4} \) of the times when flu is not actually present (this is called the false positive rate);
- currently \( \frac{1}{3} \) is the rate of the flu in the population.

For a random person, what is the probability that the flu test is positive?

**Solution and answer:**

\[
P(test+) = \frac{4}{5} \cdot \frac{1}{3} + \frac{1}{4} \cdot \frac{2}{3} = \frac{4}{15} + \frac{1}{2} \cdot \frac{2}{3} = \frac{8 + 5}{30} = \frac{13}{30}
\]

What is the probability that a random person actually has the flu, given that the flu test is positive?

**Solution and answer:**

\[
P(flu|test+) = \frac{P(flu \cap test+)}{P(test+)} = \frac{\frac{4}{15}}{\frac{13}{30}} = \frac{8}{13}
\]

(5) There are 18 cards that are divided in three groups of cards, containing 5, 6, 7 cards respectively. Each group contains 2 red cards, and the other cards are black. We shuffle all these cards and select a random card, which happens to be black. What is the probability that this particular card was from the first group?

**Solution and answer:**

the first group contains 3 back cards, and the total number of black cars is \( 18 - 6 = 12 \). Therefore the answer is \( \frac{3}{12} = \frac{1}{4} \)

Another solution is \( P(first|black) = \frac{P(first \cap black)}{P(black)} = \frac{\frac{3}{18}}{\frac{12}{18}} = \frac{1}{4} \)