MATH 3160 - Probability - Fall 2017
sample final exam *

Show all work: either write at least a sentence explaining your reasoning, or annotate your math work with brief explanations. Correct answer with no solution will give only a partial credit. There is NO need to simplify, and NO calculators are needed. • You may leave your answer in terms of sums, products, factorials or binomial coefficients, and fractions. Use the notation $\Phi(x)$ for the $\mathcal{N}(0,1)$ distribution function, that is $\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-y^2/2} dy = \mathbb{P}(Z < x)$ where Z is the standard normal random variable. You do not need a table of values of Φ.

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(1) Consider a standard deck of 52 cards. What is the probability of a four of a kind? (This occurs when the cards have denominations a, a, a, a, b.)

(2) Consider a roullete wheel consisting of 50 numbers 1 through 50, 0, and 00. If Phan always bets that the outcome will be one of the numbers 1 through 20, what is the probability that
   (a) Phan will lose his first 7 bets,
   (b) if his first win will occur on his Xth bet, what is the distribution of X? What is $\mathbb{P}(X = 9)$?

(3) The monthly worldwide average number of airplane crashes of commercial airlines is 3.5. What is the probability that at most 1 accident will occur in next 2 months?

(4) The r.v. X has a mgf given by

$$m_X(t) = \frac{1}{1-t}, \quad t < 1.$$

If u is some unknown number greater than 0, what is $\mathbb{P}(X > 1 + u \mid X > u)$?

(5) A manufacturing company sources widgets from three different suppliers (A, B, and C). Based on the company's quality control data, it appears that 3 percent of widgets coming from A are faulty, as are 5 percent of the widgets coming from B, and 2 percent coming from C. Based on recent purchasing records, suppliers A, B, and C supply 30 percent, 20 percent, and 50 percent of the company's stock of widgets, respectively.
   (a) What is the probability that a random widget from the company's stock is faulty?
   (b) Given that a widget is faulty, what is the probability that it came from supplier C?
   (c) Using the definition of independence of events, determine whether the events $F = \{\text{widget is faulty}\}$ and $C = \{\text{widget came from supplier C}\}$ are independent or not.

(6) Suppose the joint density function of the random variables X and Y is

$$f(x,y) = \begin{cases} c(x+y) & 0 < x, y < 1 \\ 0 & \text{otherwise} \end{cases}.$$

   (a) Find the value of c.
   (b) Compute $\mathbb{P}(X^2 + Y^2 \leq 1)$
   (c) Compute $\mathbb{E}[X^2 Y]$.

(7) Suppose X is a normal r.v. with mean 1 and variance 1 and let Y be an independent Poisson r.v. with parameter 2. What is $\text{Var}(2X - Y)$? What are $\mathbb{E}(2X - Y)$ and $\mathbb{E}(2X - Y)^2$?

(8) Let X be a uniform random variable over $(1, 6)$. Find the moment generating function of X. Show all steps.

(9) Suppose X has the following moment generating function

$$m_X(t) = \frac{e^t}{1 - t^2}.$$

Find $\mathbb{E}[X]$. (This distribution is known as the *Laplace* distribution)

(10) A person has 100 light bulbs whose lifetimes are independent exponentials with mean 5 hours. If the bulbs are used one at a time, with a failed bulb being replaced immediately by a new one, approximate the probability that there is still a working light bulb after 525 hours.

answer key:

(1) $\dfrac{\mathbf{13 \cdot 12 \cdot 4}}{\binom{\mathbf{52}}{\mathbf{5}}}$

(2) (a) $\left(\dfrac{\mathbf{32}}{\mathbf{52}}\right)^{\mathbf{7}}$

(b) Geometric, $\left(\dfrac{\mathbf{32}}{\mathbf{52}}\right)^{\mathbf{8}}\left(\dfrac{\mathbf{20}}{\mathbf{52}}\right)$

(3) $\boldsymbol{8e^{-7}}$

(4) $\boldsymbol{e^{-1}}$

(5) (a) $\mathbf{.01 + .01 + .009 = \dfrac{29}{1000}}$

(b) $\dfrac{\mathbf{.01}}{\mathbf{.029}} = \dfrac{\mathbf{10}}{\mathbf{29}}$

(c) no.

(6) (a) $\boldsymbol{c = 1}$

(b) $\frac{\mathbf{2}}{\mathbf{3}}$

(c) $\frac{\mathbf{1}}{\mathbf{8}} + \frac{\mathbf{1}}{\mathbf{9}} = \frac{\mathbf{17}}{\mathbf{72}}$

(7) **6**, **0** and **6**

(8) $\dfrac{\boldsymbol{e^{6t} - e^{t}}}{\mathbf{5}\boldsymbol{t}}$

(9) **1**

(10) $\mathbf{1 - \Phi(\frac{1}{2})}$