

Self-graded not-for-credit quick 8 minute Quiz

5 people, including A and B, take part in a meeting, seated in a row.

.....

- (a) How many ways can the 5 people be seated so that A and B sit together? **Ans:** $4! \cdot 2! = 48$
 - (a') That is the probability that A and B sit together? **Ans:** $\frac{4! \cdot 2!}{5!} = \frac{48}{120} = \frac{2}{5}$
 - (b) How many ways can the 5 people be seated so that A and B do NOT sit together? **Ans:** $5! - 4! \cdot 2! = 120 - 48 = 72$
 - (b') That is the probability that A and B do NOT sit together? **Ans:** $\frac{72}{120} = \frac{3}{5}$
 - (c) During this meeting, they will elect 3 of their own members to a committee. How many possible committees can be formed in which A and B serve together? **Ans:** 3, because after we select A and B, there is one place for which we can choose one of 3 people, C,D,E.
 - (c') That is the probability that A and B serve together? **Ans:** $\frac{3}{10}$, because $\binom{5}{3} = 10$ is the total number of committees.
 - (d) During this meeting, they will elect 3 of their own members to a committee. How many possible committees can be formed in which A and B do NOT serve together? **Ans:** $10 - 3 = 7$
 - (d') That is the probability that A and B do NOT serve together? **Ans:** $\frac{7}{10}$
-

We can discuss a more general situation. Suppose that there are n people, of whom k people are to be selected for a committee, and we would like to find the probability that a particular pre-selected group of r friends serve together. In this case we can select the total of $\binom{n}{k}$ different committees and, among

these choices, there are $\binom{n-r}{k-r}$ committees that contain the pre-selected group of r friends. This is because if we pre-select r friends, we will have $n-r$ people left, out of whom we need to select $k-r$. So the probability is $\frac{\binom{n-r}{k-r}}{\binom{n}{k}} = \frac{(n-r)!k!}{(k-r)!n!}$.

If we compare to question (c') with $n = 5, r = 3, k = 2$, we get the same answer.