MATH 3160 - Probability - Fall 2017
Quiz 8-9, Wednesday November 1

Show all work: either write at least a sentence explaining your reasoning, or annotate your math work with brief explanations. Correct answer with no solution will give only a partial credit. There is NO need to simplify, and NO calculators are needed. You may leave your answer in terms of sums, products, factorials or binomial coefficients, and fractions.

In this quiz use the notation $\Phi(x)$ for the distribution function for $\mathcal{N}(0,1)$, that is

$$\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-y^2/2} dy = \mathbb{P}(Z < x)$$

where Z is the standard normal random variable. You do not need a table of values of Φ.

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(1) Find a formula for $\mathbb{P}(-1 \leq X \leq 5)$ if X is $\mathcal{N}(3,4)$. Your answer should include Φ twice.

Solution: $\mathcal{N}(3,4) = \mathcal{N}(\mu, \sigma^2)$, $X = 3 + 2Z$, $\mathbb{P}(-1 \leq 3 + 2Z \leq 5) = P(-2 \leq Z \leq 1)$

Answer: $\boxed{\Phi(1) - \Phi(-2) = \Phi(1) + \Phi(2) - 1}$

(2) Find a formula for $\mathbb{P}(|X| \geq c)$ if X is $\mathcal{N}(1,9)$. Your answer should include c and Φ twice each.

Solution: $\mathcal{N}(\mu, \sigma^2)$, $X = 1 + 3Z$, if $c > 0$ then $\mathbb{P}(|1 + 3Z| \geq c) =$

$= \mathbb{P}(1+3Z \geq c) + \mathbb{P}(1+3Z \leq -c) = \mathbb{P}(Z \geq \frac{c-1}{3}) + \mathbb{P}(Z \leq \frac{-1-c}{3}) = \mathbb{P}(Z \leq \frac{1-c}{3}) + \mathbb{P}(Z \leq \frac{-1-c}{3})$

Answer: if $c > 0$ then $\boxed{\Phi(\frac{-1-c}{3}) + \Phi(\frac{1-c}{3})}$, otherwise, if $c \leq 0$, then the answer is 1.

(3) Suppose a coin is tossed 100 times. Find the formula for the best possible normal approximation for the probability to have exactly 50 heads. Your answer should include Φ twice.

Solution: $X = Bin(100, \frac{1}{2})$, $\mu = np = 50$, $\sigma = \sqrt{np(1-p)} = 5$
$\mathbb{P}(49.5 < X < 50.5) \approx \mathbb{P}(49.5 < 50 + 5Z < 50.5) = \mathbb{P}(-0.1 < z < 0.1)$

Answer: $\boxed{\Phi(0.1) - \Phi(-0.1) = 2\Phi(0.1) - 1}$

(4) Suppose a coin is tossed 100 times. Find the formula for the best possible normal approximation for the probability to have at least 40 but no more than 50 heads. Your answer should include Φ twice.

Solution: using the previous question
$\mathbb{P}(39.5 < X < 50.5) \approx \mathbb{P}(39.5 < 50 + 5Z < 50.5) = \mathbb{P}(-2.1 < Z < 0.1)$

Answer: $\boxed{\Phi(0.1) - \Phi(-2.1) = \Phi(0.1) + \Phi(2.1) - 1}$