(1) Suppose \( X \) is a Poisson random variable with parameter \( \lambda \). Find the formula for \( P(X \geq 2) \).

Answer: 

\[
P(X \geq 2) = e^{-\lambda} \sum_{i=2}^{\infty} \frac{\lambda^i}{i!} = 1 - e^{-\lambda} - \lambda e^{-\lambda}
\]

(2) Suppose \( X \) is a Poisson random variable with parameter \( \lambda \). If \( P(X = 0) = \frac{1}{2} \), can you find \( \lambda \)?

Answer: 

\[
P(X = 0) = \frac{1}{2} = e^{-\lambda}. \quad \text{Therefore} \quad \lambda = -\log \left( \frac{1}{2} \right) = \log(2)
\]

(3) Suppose that \( \frac{1}{8} \) of all students are left-handed. A class of size 24 meets in a room with 21 right-handed desks and 3 left-handed desks. What is the probability that every student can have a suitable desk?

Answer: The exact answer is 

\[
\left( \frac{24}{3} \right) \left( \frac{1}{8} \right)^3 \left( \frac{7}{8} \right)^{21} = \frac{24 \cdot 23 \cdot 22 \cdot 7^{21}}{3 \cdot 2 \cdot 8^{24}}
\]

In decimals, this is 0.239... Note that Poisson approximation with \( \lambda = 3 \) would give \( e^{-3} \frac{3^3}{3!} \approx 0.224 \), but this was not a part of the question.

(4) Suppose again that \( \frac{1}{8} \) of all students are left-handed. Another class of size 24 meets in a room with 22 right-handed desks and 4 left-handed desks. What is the probability that every student can have a suitable desk?

Answer: 

\[
\left( \frac{24}{2} \right) \left( \frac{1}{8} \right)^2 \left( \frac{7}{8} \right)^{22} + \left( \frac{24}{3} \right) \left( \frac{1}{8} \right)^3 \left( \frac{7}{8} \right)^{21} + \left( \frac{24}{4} \right) \left( \frac{1}{8} \right)^4 \left( \frac{7}{8} \right)^{20} = \sum_{k=2}^{4} \left( \frac{24}{k} \right) \left( \frac{1}{8} \right)^k \left( \frac{7}{8} \right)^{24-k}
\]

In decimals, this is 0.571...

The Poisson approximation gives \( e^{-\lambda} \sum_{i=2}^{4} \frac{\lambda^i}{i!} \approx e^{-3} \sum_{i=2}^{4} \frac{3^i}{i!} \approx 0.616 \)

(5) Suppose once again that \( \frac{1}{8} \) of all students are left-handed. Yet another class of size 24 meets in a room with 24 right-handed desks and 5 left-handed desks. What is the probability that every student can have a suitable desk?

Answer: 

\[
\sum_{k=0}^{5} \left( \frac{24}{k} \right) \left( \frac{1}{8} \right)^k \left( \frac{7}{8} \right)^{24-k}
\]

In decimals, this is 0.853...

The Poisson approximation gives \( e^{-\lambda} \sum_{i=0}^{5} \frac{\lambda^i}{i!} \approx e^{-3} \sum_{i=2}^{4} \frac{3^i}{i!} \approx 0.916 \)

Later on we will discuss the normal approximation and the so-called Z-score. Note that in this range the normal approximation becomes more accurate than the Poisson approximation. Here the Z-score value is 

\[
Z = \frac{X - \mu}{\sigma} = \frac{X - E(X)}{\sqrt{Var(X)}} = \frac{5 - 3}{\sqrt{21/8}} \approx 1.23
\]

which would gives the approximate probability 0.89