MATH 3160 - Probability - Fall 2017
Quiz 6, Wednesday October 18

(1) Suppose X is a Poisson random variable with parameter λ. Find the formula for $\mathbb{P}(X \geqslant 2)$.

Answer: $\boxed{\mathbb{P}(X \geqslant 2) = e^{-\lambda}\sum_{i=2}^{\infty} \lambda^i/i! = 1 - e^{-\lambda} - \lambda e^{-\lambda}}$

(2) Suppose X is a Poisson random variable with parameter λ. If $\mathbb{P}(X = 0) = \frac{1}{2}$, can you find λ?

Answer: $\mathbb{P}(X = 0) = \frac{1}{2} = e^{-\lambda}$. Therefore $\boxed{\lambda = -\log\left(\frac{1}{2}\right) = \log(2)}$

(3) Suppose that $\frac{1}{8}$ of all students are left-handed. A class of size 24 meets in a room with 21 right-handed desks and 3 left-handed desks. What is the probability that every student can have a suitable desk?

Answer: The exact answer is $\boxed{\binom{24}{3}\left(\frac{1}{8}\right)^3\left(\frac{7}{8}\right)^{21} = \frac{24\cdot 23\cdot 22\cdot 7^{21}}{3\cdot 2\cdot 8^{24}}}$

In decimals, this is $.239\ldots$. Note that Poisson approximation with $\lambda = 3$ would give $e^{-3}3^3/3! \approx 0.224$, but this was not a part of the question.

(4) Suppose again that $\frac{1}{8}$ of all students are left-handed. Another class of size 24 meets in a room with 22 right-handed desks and 4 left-handed desks. What is the probability that every student can have a suitable desk?

Answer: $\boxed{\binom{24}{2}\left(\frac{1}{8}\right)^2\left(\frac{7}{8}\right)^{22} + \binom{24}{3}\left(\frac{1}{8}\right)^3\left(\frac{7}{8}\right)^{21} + \binom{24}{4}\left(\frac{1}{8}\right)^4\left(\frac{7}{8}\right)^{20} = \sum_{k=2}^{4}\binom{24}{k}\left(\frac{1}{8}\right)^k\left(\frac{7}{8}\right)^{24-k}}$

In decimals, this is $0.571\ldots$

The Poisson approximation gives $e^{-\lambda}\sum_{i=2}^{4}\lambda^i/i! = e^{-3}\sum_{i=2}^{4}3^i/i! \approx 0.616$

(5) Suppose once again that $\frac{1}{8}$ of all students are left-handed. Yet another class of size 24 meets in a room with 24 right-handed desks and 5 left-handed desks. What is the probability that every student can have a suitable desk?

Answer:

$$\boxed{\sum_{k=0}^{5}\binom{24}{k}\left(\frac{1}{8}\right)^k\left(\frac{7}{8}\right)^{24-k}}$$

In decimals, this is $0.853\ldots$

The Poisson approximation gives $e^{-\lambda}\sum_{i=0}^{5}\lambda^i/i! = e^{-3}\sum_{i=2}^{4}3^i/i! \approx 0.916$

Later on we will discuss the normal approximation and the so-called Z-score. Note that in this range the normal approximation becomes more accurate than the Poisson approximation. Here the Z-score value is $Z = \frac{X-\mu}{\sigma} = \frac{X-\mathbb{E}X}{\sqrt{Var(X)}} = \frac{5-3}{\sqrt{21/8}} \approx 1.23$ which would gives the approximate probability 0.89