(1) Suppose that a fair coin is tosses 2 times, and X is the number of heads.
   (a) What is the probability mass function (pmf)? Answer: $p(0) = p(2) = \frac{1}{4}, p(1) = \frac{1}{2}$
   (b) What is the picture of the cumulative distribution function (cdf)?
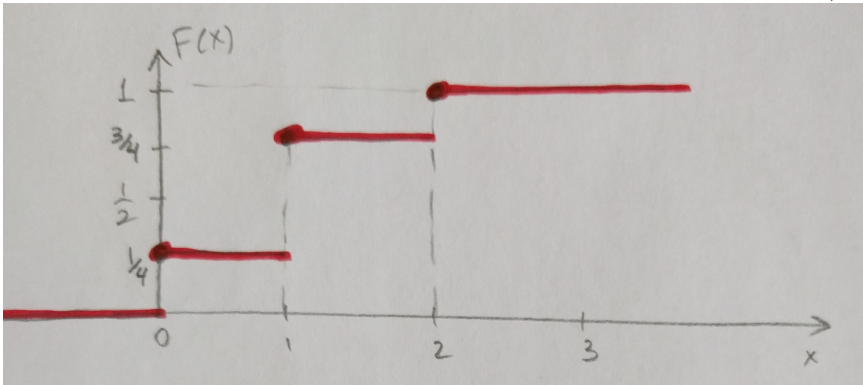
   (c) What is the definition of the cdf using cases? Answer: $F(x) = \begin{cases} 0 & \text{if } x < 0 \\ \frac{1}{4} & \text{if } 0 \leqslant x < 1 \\ \frac{3}{4} & \text{if } 1 \leqslant x < 2 \\ 1 & \text{if } 2 \leqslant x \end{cases}$

   (d) What is $\mathbb{E}X$? Answer: $\frac{1}{2} + \frac{1}{4} \cdot 2 = 1$
   (e) What is $\text{Var}(X)$? Answer: $\frac{1}{2} \cdot 1 = \frac{1}{2}$

(2) Suppose that a fair dice is tosses one time. You can win X dollars according to the following rules: if you roll a 1, then you win **$1**, if you roll a 2, then you win **$2**, if you roll a 3, then you win **$3**, otherwise you win nothing.
   (a) What is the probability mass function (pmf)? Answer: $p(0) = \frac{1}{2}, p(1) = p(2) = p(3) = \frac{1}{6}$
   (b) What is the picture of the cumulative distribution function (cdf)?
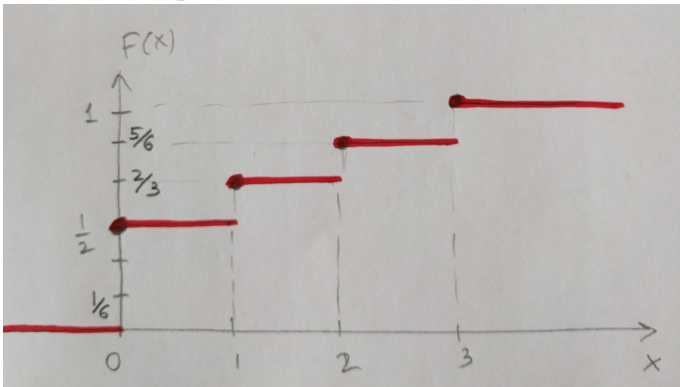
   (c) What is the definition of the cdf using cases? Answer: $F(x) = \begin{cases} 0 & \text{if } x < 0 \\ \frac{1}{2} & \text{if } 0 \leqslant x < 1 \\ \frac{2}{3} & \text{if } 1 \leqslant x < 2 \\ \frac{5}{6} & \text{if } 2 \leqslant x < 3 \\ 1 & \text{if } 3 \leqslant x \end{cases}$

   (d) What is $\mathbb{E}X$? Answer: $\frac{1}{6} \cdot 1 + \frac{1}{6} \cdot 2 + \frac{1}{6} \cdot 3 = 1$
   (e) What is $\text{Var}(X)$? Answer: $\frac{1}{6} \cdot 1 + \frac{1}{6} \cdot 4 + \frac{1}{6} \cdot 9 - 1 = \frac{7}{3} - 1 = \frac{4}{3}$