MATH 3160 - Probability - Fall 2017
Quiz 4, Wednesday September 27

(1) Suppose that, among **10** families, **4** of families own a dog, **3** of families own a cat, and **4** of the families own neither. How many families own both a cat and a dog?

Solution: There are $\mathbf{10-4=6}$ families who own $\mathbf{3+4=7}$ animals. Therefore **1** family owns a dog and a cat because $\mathbf{7-6=1}$.

This is the contingency table:

	a dog	no dog	total
a cat	1	2	3
no cat	3	4	7
total	4	6	10

(2) A family is chosen at random among the same **10** families as in the previous problem, and found to have a dog. What is the probability they also own a cat?

Solution: $\mathbb{P}(\boldsymbol{cat}|\boldsymbol{dog}) = \dfrac{\mathbb{P}(\boldsymbol{cat\ and\ dog})}{\mathbb{P}(\boldsymbol{dog})} = \dfrac{\mathbf{1}}{\mathbf{4}}$

(3) In a multiple choice test, a student either knows the answer to a question or gives a random answer. Each question has **4** possible answers, and the student knows the answer to a question with probability $\frac{\mathbf{2}}{\mathbf{3}}$. Find the probability that the student knows the answer to a question, given that the answer was correct.

Solution: If the student Knows the answer, this answer is Correct.

Hence we have $\boldsymbol{K} \cap \boldsymbol{C} = \boldsymbol{K}$, and therefore $\mathbb{P}(\boldsymbol{K}|\boldsymbol{C}) = \dfrac{\mathbb{P}(\boldsymbol{K} \cap \boldsymbol{C})}{\mathbb{P}(\boldsymbol{C})} = \dfrac{\frac{2}{3}}{\frac{2}{3} + \frac{1}{3} \cdot \frac{1}{4}} = \dfrac{\mathbf{8}}{\mathbf{9}}$

Suppose that

- a flu test (correctly) indicates the presence of the flu $\frac{\mathbf{9}}{\mathbf{10}}$ of the times when the patient actually has the flu *(this is called the true positive rate)*;
- the same test (incorrectly) indicates the presence of flu $\frac{\mathbf{3}}{\mathbf{10}}$ of the times when flu is not actually present *(this is called the false positive rate)*;
- currently $\frac{\mathbf{1}}{\mathbf{3}}$ of the population has the flu.

(4) For a random person, what is the probability that the flu test is positive?

Solution:

$$\mathbb{P}(\{\boldsymbol{test+}\}) = \mathbb{P}(\{\boldsymbol{flu}\} \cap \{\boldsymbol{test+}\}) + \mathbb{P}(\{\boldsymbol{no\ flu}\} \cap \{\boldsymbol{test+}\}) = \frac{\mathbf{1}}{\mathbf{3}} \cdot \frac{\mathbf{9}}{\mathbf{10}} + \frac{\mathbf{2}}{\mathbf{3}} \cdot \frac{\mathbf{3}}{\mathbf{10}} = \frac{\mathbf{15}}{\mathbf{30}} = \frac{\mathbf{1}}{\mathbf{2}}$$

(5) Calculate the probability that a random person actually has the flu, given that the flu test is positive.

Solution: $\mathbb{P}(\boldsymbol{flu}|\boldsymbol{test+}) = \dfrac{\mathbb{P}(\{\boldsymbol{flu}\} \cap \{\boldsymbol{test+}\})}{\mathbb{P}(\{\boldsymbol{test+}\})} = \dfrac{\frac{1}{3} \cdot \frac{9}{10}}{\frac{1}{2}} = \dfrac{\mathbf{9}}{\mathbf{15}} = \dfrac{\mathbf{3}}{\mathbf{5}}$