(1) Suppose that, among 10 families, 4 of families own a dog, 3 of families own a cat, and 4 of the families own neither. How many families own both a cat and a dog?

**Solution:** There are $10 - 4 = 6$ families who own $3 + 4 = 7$ animals. Therefore 1 family owns a dog and a cat because $7 - 6 = 1$.

This is the contingency table:

<table>
<thead>
<tr>
<th></th>
<th>a dog</th>
<th>no dog</th>
<th>total</th>
</tr>
</thead>
<tbody>
<tr>
<td>a cat</td>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>no cat</td>
<td>3</td>
<td>4</td>
<td>7</td>
</tr>
<tr>
<td>total</td>
<td>4</td>
<td>6</td>
<td>10</td>
</tr>
</tbody>
</table>

(2) A family is chosen at random among the same 10 families as in the previous problem, and found to have a dog. What is the probability they also own a cat?

**Solution:** $P(\text{cat}|\text{dog}) = \frac{P(\text{cat and dog})}{P(\text{dog})} = \frac{1}{4}$

(3) In a multiple choice test, a student either knows the answer to a question or gives a random answer. Each question has 4 possible answers, and the student knows the answer to a question with probability $\frac{2}{3}$. Find the probability that the student knows the answer to a question, given that the answer was correct.

**Solution:** If the student knows the answer, this answer is correct. Hence we have $K \cap C = K$, and therefore $P(K|C) = \frac{P(K \cap C)}{P(C)} = \frac{\frac{2}{3}}{\frac{2}{3} + \frac{1}{3} \cdot \frac{1}{4}} = \frac{8}{9}$

Suppose that
- a flu test (correctly) indicates the presence of the flu $\frac{9}{10}$ of the times when the patient actually has the flu (*this is called the true positive rate*);
- the same test (incorrectly) indicates the presence of flu $\frac{3}{10}$ of the times when flu is not actually present (*this is called the false positive rate*);
- currently $\frac{1}{3}$ of the population has the flu.

(4) For a random person, what is the probability that the flu test is positive?

**Solution:**

$$P(\{\text{test}+\}) = P(\{\text{flu} \cap \{\text{test}+\}\}) + P(\{\text{no flu} \cap \{\text{test}+\}\}) = \frac{1}{3} \cdot \frac{9}{10} + 2 \cdot \frac{3}{10} = \frac{15}{30} = \frac{1}{2}$$

(5) Calculate the probability that a random person actually has the flu, given that the flu test is positive.

**Solution:** $P(\text{flu}|\text{test}+) = \frac{P(\{\text{flu} \cap \{\text{test}+\}\})}{P(\{\text{test}+\})} = \frac{\frac{1}{3} \cdot \frac{9}{10}}{\frac{1}{2}} = \frac{9}{15} = \frac{3}{5}$