

MATH 3160 - Probability - Fall 2017
Quiz 3, Wednesday September 20

Show all work. You should either write at least a sentence explaining your reasoning, OR annotate your math work with brief explanations. Correct answer with no solution will give only a partial credit. You may leave your answer in terms of sums, products, factorials or binomial coefficients, and fractions. There is NO need to simplify. NO calculators are needed.

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- (1) Two dice are simultaneously rolled. Compute if the following two events are independent or not.

$$A = \{\text{the sum is 7}\}, B = \{\text{the second die lands a 5}\}.$$

Answer: independent because $P(A) = P(B) = \frac{1}{6}$ and $P(A \cap B) = \frac{1}{36}$.

- (2) Two dice are simultaneously rolled. Compute if the following three events are independent or not.

$$A = \{\text{the sum is 7}\}, B = \{\text{the second die lands a 5}\}, C = \{\text{the first die lands a 2}\}.$$

Answer: *not* independent because $P(A) = P(B) = P(C) = \frac{1}{6}$,

but $P(A \cap B \cap C) = P(A \cap B) = \frac{1}{36} \neq \frac{1}{6^3}$.

- (3) Suppose you toss a fair coin repeatedly and independently. If it comes up heads, you win a dollar, and if it comes up tails, you lose a dollar. Suppose you start with \$4. What is the probability you will get up to \$6 before you go broke?

Answer: $\frac{2}{3}$. Probability $y(x)$ to win with initial amount x is a linear function such as $y(0) = 0$, $y(6) = 1$. Hence $y(x) = \frac{x}{6}$.

- (4) 3 balls are taken at random from a black box that contained 4 Green and 4 Blue balls. What is the probability that all balls are of the same color?

Answer: $\frac{1}{7}$. This is the same as $\frac{2 \cdot \binom{4}{3}}{\binom{8}{3}}$. Another solution is by the multiplication rule $\frac{3}{7} \cdot \frac{2}{6}$.

- (5) 3 balls are taken at random from a black box that contained 4 Green and 4 Blue balls. What is the probability that exactly 1 of the balls is Green?

Answer: $\frac{3}{7}$. This is the same as $\frac{\binom{4}{1} \binom{4}{2}}{\binom{8}{3}}$. Another solution is by the multiplication rule $3 \cdot \frac{4}{8} \cdot \frac{4}{7} \cdot \frac{3}{6}$.

Extra credit question: if x is the probability in problem 4, and y is the probability in problem 5, what is $x + 2y$? Hint: you do not need to perform any computations to answer this question, but an explanation is needed.

Answer: $x + 2y = 1$ because this is the total probability. Either all colors are the same, or there is one ball of one color (either Green or Blue), and two balls of the other color. Probabilities of these three events are x, y, y , they are disjoint, and together cover all possibilities.