MATH 3160 - Probability - Fall 2017
Quiz 11-12, Wednesday November 29

Consider the situation when c is a positive number and X, Y are random variables with this joint p.d.f.:

$$f(x,y) = \begin{cases} c\,(2x+3y) & 0 \le x \le 1, 0 \le y \le 1 \\ 0 & \text{otherwise.} \end{cases}$$

(1) Find c

Solution: First, compute $\int_0^1 \int_0^1 (2x+3y)dxdy = \frac{5}{2}$. Then, note that $\int_0^1 \int_0^1 f(x,y)dxdy = 1$ because this is the total probability. Hence, c is 1 divided by the first integral.

Answer:

$$c = \frac{2}{5}$$

(2) Find $\mathbb{E}Y$

Solution: We can compute the marginal density $f_Y(y) = \int_0^1 f(x,y)dx = \frac{2}{5} + \frac{6}{5}y$

Answer:

$$\mathbb{E}Y = \int_0^1 y f_Y(y)dy = \int_0^1 y(\tfrac{2}{5} + \tfrac{6}{5}y)dy = \tfrac{1}{5} + \tfrac{2}{5} = \frac{3}{5}$$

(3) Find $\mathbb{E}X$, but do not simplify fractions.

Solution: We can compute the marginal density $f_X(x) = \int_0^1 f(x,y)dy = \frac{4}{5}x + \frac{3}{5}$

Answer:

$$\mathbb{E}X = \int_0^1 x f_X(x)dx = \int_0^1 x(\tfrac{4}{5}x + \tfrac{3}{5})dx = \tfrac{4}{15} + \tfrac{3}{10} = \frac{17}{30}$$ the simplification was not required

(4) Find $\mathbb{E}(XY)$, but do not simplify fractions.

Solution: $\int_0^1 \int_0^1 xyf(x,y)dxdy = \frac{2}{5}\int_0^1 \int_0^1 (2x^2y + 3xy^2)dxdy = \frac{2}{5}(2+3)\frac{1}{2}\frac{1}{3}$

Answer:

$$\mathbb{E}(XY) = \frac{1}{3}$$ here the simplification was not required

(5) Are X, Y independent? Explain briefly.

Solution: We can not write $2x+3y$ as a product of two functions, that is $f(x,y) \neq f_X(x)f_Y(y)$. Note also that $\mathbb{E}(XY) \neq \mathbb{E}(X)\mathbb{E}(Y)$

Answer:

Not independent.