Laplace transforms: $F(s) = \mathcal{L}\{f\}(s) = \int_0^\infty e^{-st} f(t)dt$

$$\mathcal{L}\{1\}(s) = \frac{1}{s}$$

$$\mathcal{L}\{t^n\}(s) = \frac{n!}{s^{n+1}}$$

$$\mathcal{L}\{e^{at}\}(s) = \frac{1}{s-a}$$

$$\mathcal{L}\{\cos bt\}(s) = \frac{s}{s^2+b^2} \qquad \mathcal{L}\{t\ \cos bt\}(s) = \frac{s^2-b^2}{(s^2+b^2)^2}$$

$$\mathcal{L}\{\sin bt\}(s) = \frac{b}{s^2+b^2} \qquad \mathcal{L}\{t\ \sin bt\}(s) = \frac{2bs}{(s^2+b^2)^2}$$

$$\mathcal{L}\{f'\}(s) = s\mathcal{L}\{f\}(s) - f(0)$$

$$\mathcal{L}\{f''\}(s) = s^2\mathcal{L}\{f\}(s) - sf(0) - f'(0)$$

$$\mathcal{L}\{e^{at}f(t)\}(s) = F(s-a)$$

Heaviside step function:

$$u(t) = \begin{cases} 1, t > 0 \\ 0, t < 0 \end{cases} \qquad u(t-a) = \begin{cases} 1, t > a \\ 0, t < a \end{cases}$$

$$\mathcal{L}\{u(t-a)f(t-a)\}(s) = e^{-as}F(s)$$

$$\mathcal{L}^{-1}\{e^{-as}F(s)\}(t) = u(t-a)f(t-a)$$

$$\mathcal{L}\{u(t-a)\}(s) = \frac{e^{-as}}{s}$$

$$u_a(t) = u(t-a)$$

Dirac delta function: $\delta_a = u_a'$ $\int_0^\infty \delta_a(t)f(t)dt = f(a)$ $\boxed{\mathcal{L}\{\delta_a\}(s) = e^{-as}}$

Periodic delta forcing:

$$\mathcal{L}\{\sum_{n=1}^{\infty} \delta_{na}\}(s) = \sum_{n=1}^{\infty} e^{-ans} = \sum_{n=1}^{\infty} \left(e^{-as}\right)^n = \frac{e^{-as}}{1-e^{-as}}$$