Basic Rules of Algebra

Included below are many of the basic rules for manipulating arithmetic and algebraic expressions. If you cannot justify a calculation you've done based on these rules, then it is probably incorrect.

Thus, it is necessary to be thoroughly familiar with every one of these rules.

Basic Properties of Real Numbers

Commutative Laws: $a + b = b + a$, $a \cdot b = b \cdot a$

Associative Laws: $(a + b) + c = a + (b + c)$, $(ab)c = a(bc)$

Distributive Law: $a(b \pm c) = ab \pm ac$

Cancellation Law: If $c \neq 0$ then $\frac{ac}{bc} = \frac{a}{b}$

An important consequence of the Cancellation Law is that the only way a product of two numbers can equal 0 is if at least one of the factors is 0. This is the key to solving most equations.

Properties of Exponents

$a^0 = 1$ $\quad a^{-n} = \frac{1}{a^n}$

$a^r a^s = a^{r+s}$ $\quad a^{r-s} = \frac{a^r}{a^s}$

$(ab)^r = a^r b^r$ $\quad \left(\frac{a}{b}\right)^r = \frac{a^r}{b^r}$

$(a^r)^s = a^{rs}$

Properties of Radicals

De nition of a Fractional Exponent: $a^{m/n} = (\sqrt[n]{a})^m = \sqrt[n]{a^m}$

$\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$ $\quad \sqrt[n]{\frac{a}{b}} = \frac{\sqrt[n]{a}}{\sqrt[n]{b}}$ $\quad \sqrt[n]{\sqrt[m]{x}} = \sqrt[mn]{x}$

Properties of Rational Expressions

$\frac{a}{d} + \frac{b}{d} = \frac{a+b}{d}$ $\quad \frac{a}{b} \cdot \frac{c}{d} = \frac{ac}{bd}$

$\frac{a}{b} = a \cdot \frac{1}{b} = a \div b$ $\quad \frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \cdot \frac{d}{c} = \frac{ad}{bc}$

Properties of Absolute Value

De nition of Absolute Value: $|a| = \begin{cases} a & \text{if } a \geq 0, \\ -a & \text{if } a < 0 \end{cases}$

$|ab| = |a| \cdot |b|$ $\quad \left|\frac{a}{b}\right| = \frac{|a|}{|b|}$

Triangle Inequality: $|a + b| \leq |a| + |b|$

$|a - b| =$ the distance between a and b on a number line

Properties of Inequalities

$a < b$ if and only if $a + c < b + c$ $\quad a < b$ if and only if $a - c < b - c$
$a > b$ if and only if $a + c > b + c$ $\quad a > b$ if and only if $a - c > b - c$
If $c > 0$ then $a < b$ if and only if $ac < bc$ and $a < b$ if and only if $a/c < b/c$
If $c > 0$ then $a > b$ if and only if $ac > bc$ and $a > b$ if and only if $a/c > b/c$
If $c < 0$ then $a < b$ if and only if $ac > bc$ and $a < b$ if and only if $a/c > b/c$
If $c < 0$ then $a > b$ if and only if $ac < bc$ and $a > b$ if and only if $a/c < b/c$

In other words, multiplying or dividing by a negative number changes the sense of an inequality while multiplying or dividing by a positive number leaves the sense of the inequality as it was.

Properties of Logarithms