

Polynomial Multiplication

©Alan H. Stein

The University of Connecticut at Waterbury

stein@math.uconn.edu <http://www.math.uconn.edu/~stein>

A polynomial is a sum (or difference) of terms, each of which is a constant times a non-negative integral power of a variable (usually, but not always, denoted by the letter x). For example, $7x^5 + 3x^2 - 7$ or $x^23 - 5x^8 + 14x^2 - 9$.

In the first example, the terms are $7x^5$, $3x^2$ and -7 , while in the second the terms are x^23 , $-5x^8$, $14x^2$ and -9 .

(Note: You may look at the last term of the first polynomial and think of subtracting the last term, 7, or adding the last term, -7 . Either is correct, but it is probably simpler to think of terms with minus signs in the latter way even though they are written in the former way.)

The rules for the multiplication of polynomials are a consequence of the commutative, associative and distributive laws of arithmetic. In a nutshell, they boil down to the following:

To multiply two polynomials, individually multiply each term of the first by each term of the second and then add all the individual products together.

That's all there is to it. There is no need to remember special cases or special rules, like the infamous FOIL method, that work for some polynomials but not all. Indeed, the above method works for all algebraic expressions which are sums of terms, not just polynomials.

Examples

Consider the product $(2x - 3) \cdot (5x + 9)$. The first factor has two terms, $2x$ and -3 , while the second factor also has two terms, $5x$ and 9 .

Start by pairing up the first term of the first factor, $2x$, with each term of the second factor, $5x$ and 9 . This gives two products, $2x \cdot 5x$ and $2x \cdot 9$.

Next, pair up the second term of the first factor, -3 , with each term of the second factor, $5x$ and 9 . This also gives two products, $(-3) \cdot 5x$ and $(-3) \cdot 9$.

Multiplying out each individual product, we get the following four products: $10x^2$, $18x$, $-15x$ and -27 .

Now add them together: $10x^2 + 18x + (-15x) + (-27)$, and combine like terms to get $10x^2 + 3x - 27$.

Multiplying out polynomials with more terms is no harder{it's just longer. For a final example, consider the product $(x^4 - 3x^2 + 9) \cdot (x^2 + 3)$.

Pairing terms and multiplying:

$$x^4 \cdot x^2 = x^6$$

$$x^4 \cdot 3 = 3x^4$$

$$(-3x^2) \cdot x^2 = -3x^4$$

$$(-3x^2) \cdot 3 = -9x^2$$

$$9 \cdot x^2 = 9x^2$$

$$9 \cdot 3 = 27$$

Adding these terms together we get: $x^6 + 3x^4 + (-3x^4) + (-9x^2) + 9x^2 + 27$. Combining like terms, we wind up with the product $x^6 + 27$.