Mathematics 227 Name: ____________________

Professor Alan H. Stein

Due Monday, December 10, 2001 This problem set is worth 50 points.

If this problem set is handed in by Wednesday, December 5, it will be graded and returned on Monday, December 10.

Make sure that you check the course website for instructions and ll out the pledge form and hand it in with your paper. Remember that your paper may be handed in before the deadline but that no late papers will be accepted regardless of the reason. The course website also includes an explanation of how your average will be calculated if you fail to complete this assignment.

Note that, since many of the questions in this problem set can be done routinely using Maple or Mathematica, it will obviously be necessary to show, through your work, exactly how you came up with your solutions.

(1) For each vector space, nd its dimension and a basis.
  (a) The set of 3×2 matrices.
  (b) The set of vectors from the origin to points in the plane $2x + 3y - 4z = 0$.
  (c) $\{f | f : \mathbb{R} \to \mathbb{R},\ f'(x) = 3f(x)\}$.
  (d) The set of polynomials of degree 4 or less with no cubic term.
  (e) The column space of the matrix $\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 2 \\ -1 & 1 & -1 \\ -1 & 2 & 0 \end{bmatrix}$.

(2) Consider the linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^2$ given by $T(< a, b, c >) = < a + b, b - c >$.
  (a) Find the dimension of and a basis for the kernel of T.
  (b) Find the dimension of and a basis for the range of T.
  (c) Find the standard matrix for T.
  (d) Find the matrix for T if the basis for $\mathbb{R}^2$ is taken as $\{< 2, 1 >, < 1, 2 >\}$ rather than the standard basis.

(3) Let $V = \mathcal{P}_3 = \{ax^3 + bx^2 + cx + d\}$ under the usual operations for polynomials with basis $\mathcal{B} = \{1,\ 1 + x,\ 1 + x + x^2,\ 1 + x + x^2 + x^3\}$, let $D : V \to V$ be the di erentiation operator and let $T : V \to V$ be de ned by $T(ax^3 + bx^2 + cx + d) = bx^3 + cx^2 + dx + a$.
  (a) Verify that T is a linear operator.
  (b) Find the matrix for D using the given basis.
  (c) Find the matrix for T using the given basis.
  (d) Determine whether D is 1-1.
  (e) Determine whether T is 1-1.
  (f) Determine whether D is onto.
  (g) Determine whether T is onto.
  (h) Let $\mathcal{B}_1 = \{1, x, x^2, x^3\}$. Find the change-of-coordinates matrix $\underset{\mathcal{B}_1 \leftarrow \mathcal{B}}{P}$.

(4) Consider the linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ given by the matrix $\begin{bmatrix} 3 & 1 \\ 1 & 3 \end{bmatrix}$.

(a) Find $T(< 5, -2 >)$.

(b) Solve the characteristic equation $|T - \lambda I)| = 0$, where I is the identity matrix. *The solutions are the eigenvalues for T. Denote them by λ_1 and λ_2.*

(c) Find vectors $\mathbf{v}_i$ such that $T(\mathbf{v}_i) = \lambda_i \mathbf{v}_i$ for $i = 1, 2$. *These vectors are the eigenvectors for T.*

(d) Let $\mathcal{B} = \{\mathbf{v}_1, \mathbf{v}_2\}$ and nd the matrix representation for T with respect to the basis $\mathcal{B}$.