Mathematics 2110
Problem Set
Due Thursday, December 4, 2008

Print Name: ____________________

Signature: ____________________

Your signature is your pledge that you have adhered to the guidelines for problem sets and take-home examinations.

This problem set will be graded on the basis of 100 points but will be worth 50 points. Make sure that you check the course website for instructions, available from the *General Policies* portion of the web site. Remember that your paper may be handed in before the deadline but that no late papers will be accepted regardless of the reason. The course website also includes an explanation of how your average will be calculated if you fail to complete this assignment.

Note that, since most of the calculations involved can be done routinely using either a calculator or a symbolic manipulation program such as Maple or Mathematica, it will obviously be necessary to show, through your work, exactly how you came up with your solutions.

If this problem set it submitted by the next to the last class of the semester, every effort will be made to have it graded and returned at the last class.

Each question will be given the same weight. Please don't omit the extra credit question.

1. Evaluate $\iint_{\mathcal{D}} (x+y)e^{x^2-y^2}\,dA$, where $\mathcal{D}$ is the parallelogram bounded by the lines $x-y=0$, $x-y=2$, $x+y=0$, $x+y=3$. *Hint: Use a change of variables.*
2. Determine whether $\mathbf{F}(x,y) = (2x\sin y - \sin x)\mathbf{i} + (x^2\cos y)\mathbf{j}$ is a conservative vector field. If it is a conservative vector field, find a potential function $\phi(x,y)$ such that $\nabla\phi = \mathbf{F}$.
3. Calculate $\int_{\mathcal{C}} y\,dx + z\,dy + x\,dz$, where $\mathcal{C}$ is the line segment from the origin to the point $(1,1,1)$.
4. Calculate $\int_{\mathcal{C}} y\,dx + z\,dy + x\,dz$, where $\mathcal{C}$ is the union of the line segment going from the origin to the point $(1,0,0)$ to the point $(1,1,0)$ to the point $(1,1,1)$.
5. A thin wire is bent into the shape of a semicircle $x^2+y^2=4$, $y \geq 0$. If the linear density is $k|x|y$, for some constant $k>0$, find the mass and center of mass of the wire.
6. Calculate $\int_{\mathcal{C}} (\ln y + 2xy^3)\,dx + (3x^2y^2 + x/y)\,dy$, where $\mathcal{C}$ is the path from $(0,1)$ to $(2,5)$ along the parabola $y = x^2+1$.
7. Use a line integral to show the area of a unit circle is π.
8. Use Green's Theorem to evaluate $\int_{\mathcal{C}} \sin y\,dx + x\cos y\,dy$, where $\mathcal{C}$ is the ellipse $x^2+xy+y^2=1$, oriented counterclockwise.
9. Find a parametrization of the unit sphere with center at the origin.

10. Find a parametrization of the ellipsoid $\frac{x^2}{4} + \frac{y^2}{9} + \frac{z^2}{25} = 1$.

11. Find a parametrization of a cone of radius 5 and height 20. *Hint: It will probably be easiest if you orient the cone with its vertex at the origin and hold it the way you'd hold an ice cream cone if you didn't want the ice cream to fall out of the cone.*

12. Find a parametrization of a cylinder of height 20 and radius 5.

13. Evaluate the surface integral $\iint\limits_S xyz\,dS$, where S is the triangular region with vertices $(1, 0, 0)$, $(0, 1, 0)$, $(0, 0, 1)$.

14. Use Stokes' Theorem to evaluate $\int_{\mathcal{C}} e^{-x}\,dx + e^x\,dy + e^z\,dz$, where $\mathcal{C}$ is the triangle with vertices $(1, 0, 0)$, $(0, 2, 0)$, $(0, 0, 1)$, oriented counterclockwise when viewed from above.

Extra Credit

Extra credit will be awarded for the best joke. All jokes must observe standards of good taste.