Exact Differential Equations

Test for exactness and solve.

1. $x + y + (x - y)\dfrac{dy}{dx} = 0$
2. $6x + y^2 + y(2x - 3y)\dfrac{dy}{dx} = 0$
3. $y^2 - 2xy + 6x - (x^2 - 2xy + 2)\dfrac{dy}{dx} = 0$
4. $\cos 2y - 3x^2y^2 + (\cos 2y - 2x \sin 2y - 2x^3y)\dfrac{dy}{dx} = 0$
5. $(r + \sin\theta - \cos\theta)\dfrac{dr}{d\theta} + r(\sin\theta + \cos\theta) = 0$
6. $\sin\theta - 2r\cos^2\theta + r\cos\theta(2r\sin\theta + 1)\dfrac{d\theta}{dr} = 0$
7. $2xy + (x^2 + y^2)\dfrac{dy}{dx} = 0$

Linear Differential Equations
Distinct Roots

Find the general solutions of the following differential equations.

8. $y'' + 2y' - 3y = 0$
9. $y'' - 5y' + 6y = 0$
10. $y''' + 3y'' - 4y' = 0$
11. $y''' - 3y'' - 10y' = 0$
12. $y''' + 6y'' + 11y' + 6y = 0$
13. $y''' + y'' - 2y' = 0$

Find the solution of the differential equation satisfying the given initial conditions.

14. $y'' - 2y' - 3y = 0$, $y = 4$ and $y' = 0$ when $x = 0$.

15. $y'' - y' - 6y = 0$, $y = 3$ and $y' = -1$ when $x = 0$.

Repeated Roots

Find the general solution.

16. $y'' - 6y' + 9y = 0$

17. $y'' + 4y' + 4 = 0$

18. $4y''' + 4y'' + y' = 0$

19. $y^{(5)} - y^{(3)} = 0$

20. $y^{(5)} + y^{(3)} = 0$

21. $y^{(4)} - 2y''' - 3y'' + 4y' + 4y = 0$

Find the particular solution.

22. $y'' + 4y' + 4y = 0$, $y = 1$ and $y' = -1$ when $x = 0$

23. $y'' + 4y' + 4y = 0$, $y = 2$ when $x = 0$ and $y = 1$ when $x = 2$.

Imaginary Roots

Solve the differential equations.

24. $y'' - 2y' + 2y = 0$

25. $y'' + 9y = 0$

26. $y'' + 4y' + 5y = 0$

27. $y'' + 18y' + 81y = 0$

28. $y^{(4)} + 18y'' + 81y = 0$