Mathematics 211 Name: _____

Professor Alan H. Stein

Due Wednesday, April 26 This problem set is worth 50 points.

Make sure that you check the course website for instructions, fill out the pledge form and hand it in with your paper. The instructions for problem sets and take-home examinations along with the pledge form are available from the *General Policies* portion of the web site. *No paper will be accepted without a signed pledge form.* Remember that your paper may be handed in before the deadline but that no late papers will be accepted regardless of the reason. The course website also includes an explanation of how your average will be calculated if you fail to complete this assignment.

Note that, since most of the calculations involved can be done routinely using either a calculator or a symbolic manipulation program such as Maple or Mathematica, it will obviously be necessary to show, through your work, exactly how you came up with your solutions.

1. Find the general solution of $\frac{d^2x}{dt^2} - 16x = 0$.
2. Find the general solution of $\frac{d^2x}{dt^2} + 16x = 0$.
3. Find the general solution of $\frac{d^2x}{dt^2} - 4\frac{dx}{dt} + 4x = 6e^t$.
4. Find the general solution of $\frac{d^3x}{dt^3} - 6\frac{d^2x}{dt^2} + 11\frac{dx}{dt} - 6x = 0$.
5. A spring is such that a 4 pound weight stretches the spring 0.4 feet. The 4 pound weight is attached to the spring and the weight is started from the equilibrium position with an initial upward velocity of 2 feet per second.
   (a) Set up a differential equation to model this.
   (b) Solve the differential equation.
   (c) Describe the motion of the weight.
6. Repeat the previous question with the added condition that the motion takes place in a medium which furnishes a retarding force of a magnitude numerically equall to the speed of the weight (in feet per second).

7. Find the general solution of the system:

$$\begin{aligned} \frac{dx}{dt} &= 4x - y \\ \frac{dy}{dt} &= 2x + y \end{aligned}$$

8. Use the definition of a Laplace Transform to derive the formula $\mathcal{L}[\sin \omega t] = \dfrac{\omega}{s^2 + \omega^2}$.

9. Suppose $a, b > 0$. Derive the formula $a \sin\theta + b \cos\theta = \sqrt{a^2 + b^2} \sin(\theta + \delta)$, where $\delta = \arccos\left(\dfrac{a}{\sqrt{a^2 + b^2}}\right)$.