Mathematics 211 Name: ____________________
Professor Alan H. Stein
Due Monday, February 27 This problem set is worth 50 points.

Make sure that you check the course website for instructions, fill out the pledge form and hand it in with your paper. The instructions for problem sets and take-home examinations along with the pledge form are available from the *General Policies* portion of the web site. *No paper will be accepted without a signed pledge form.* Remember that your paper may be handed in before the deadline but that no late papers will be accepted regardless of the reason. The course website also includes an explanation of how your average will be calculated if you fail to complete this assignment.

Note that, since most of the calculations involved can be done routinely using either a calculator or a symbolic manipulation program such as Maple or Mathematica, it will obviously be necessary to show, through your work, exactly how you came up with your solutions.

1. Set up a differential equation of the form $\frac{dP}{dt} = f(t, P)$, where P represents population and t represents time, assuming a logistic population model, where in the year 2000 the population is $2.5 \cdot 10^8$ (250 million), is growing at an annual rate of $4 \cdot 10^6$ (4 million), and the ecological system can sustain a maximum population of 10^9 (one billion).

(2-10) Consider the differential equation $\frac{dx}{dt} = \frac{x}{25}\left(1 - \frac{x}{10^6}\right)$.

2. Sketch the slope field for the differential equation. *You may submit a graph created by a computer program.*

3. Since this differential equation is autonomous, sketch the phase line.

4. Use the phase line to sketch the solutions of the differential equation. Describe the different classes of solutions in relatively plain language.

5. List the equilibrium points and classify each one as a source, sink or node.

6. Obtain the general solution for the differential equation.

(7-10) Consider the same differential equation but with initial condition $x = 8 \cdot 10^6$ when $t = 0$.

7. Find the solution of the differential equation satisfying the given initial conditions.

8. Determine $\lim_{t\to\infty} x$. Explain how this fits in with your answer to question 4.

9. Use Euler's Method to get a numerical (approximate) solution to the differential equation. Let $\Delta t = 10$ and find $x_0, x_1, x_2, \ldots, x_{10}$.

10. Compare your numerical solution for $t = 100$ to the exact solution. What are the absolute and relative errors?

11. Sketch the phase line for the differential equation $\dfrac{dy}{dt} = y^3 - 16y^2$. Identify each equilibrium point as a skin, source or node.

12. Find the general solution for the differential equation $\dfrac{dy}{dx} = \dfrac{2x}{y + x^2 y}$. Find the particular solution satisfying the initial condition $y = 3$ when $x = 2$.

(13-16) A room containing 1000 cubic feet of air is originally free of carbon monoxide. The ventilation system starts blowing smoke into the room at a rate of 0.1 cubic feet per minute. The smoke contains a 4 percent concentration of carbon monoxide and is immediately completely mixed with the air in the room, and the well-circulated mixture leaves the room at the same rate the smoke is blown into the room.

13. Set up a model for this situation. Clearly list all relevant variables, along with their meaning. Your model should consist of a differential equation with an initial condition.

14. Solve the differential equation you set up.

15. How long will it take for the concentration of carbon monoxide to reach 1 percent?

16. In the long-term, what level will the concentration of carbon-monoxide approach?