

Mathematics 210

Professor Alan H. Stein

Due Monday, November 26, 2007

Name: _____

This problem set is worth 50 points.

Make sure that you check the course website for instructions, fill out the pledge form and hand it in with your paper. The instructions for problem sets and take-home examinations along with the pledge form are available from the *General Policies* portion of the web site. *No paper will be accepted without a signed pledge form.* Remember that your paper may be handed in before the deadline but that no late papers will be accepted regardless of the reason. The course website also includes an explanation of how your average will be calculated if you fail to complete this assignment.

Note that, since most of the calculations involved can be done routinely using either a calculator or a symbolic manipulation program such as Maple or Mathematica, it will obviously be necessary to show, through your work, exactly how you came up with your solutions.

Each question will be given the same weight.

1. Consider $\iiint_{\mathcal{D}} x^2 + 3xz \, dV$, where \mathcal{D} is the solid bounded by the graphs of $x^2 + y^2 - z = -1$ and $x^2 + y^2 + z = 37$. Sketch the region \mathcal{D} and write the triple integral as an iterated integral using rectangular coordinates.
2. Write the triple integral in the previous question as an iterated integral using cylindrical coordinates.
3. William Tell's son places a perfectly spherical apple with a radius of 5 centimeters on his head. William Tell shoots an arrow with a radius of a half centimeter directly through the center of the apple. What is the volume of the remaining portion of the apple?
4. Consider a tetrahedron with vertices $(1, 0, 0)$, $(0, 1, 0)$, $(-1, 0, 0)$, $(0, -1, 0)$ and $(0, 0, 2)$ and density $\delta(x, y, z) = |x| + |y| + |z|$. Sketch the tetrahedron and find its volume, mass, all three first moments and its center of mass. *It should be obvious that two of the moments and two of the coordinates of the center of mass will equal 0, but just saying it's obvious will obviously not suffice.*
5. Calculate $\frac{\partial(x, y, z)}{\partial(\rho, \theta, \phi)}$ if $x = \rho \sin \phi \cos \theta$, $y = \rho \sin \phi \sin \theta$ and $z = \rho \cos \phi$.
6. Let \mathcal{D} be the region bounded by the x -axis and the lines $x - y = 2$, $y = 1$ and $x = y$. Sketch \mathcal{D} . Calculate $\iint_{\mathcal{D}} x^2 + 2xy \, dA$ using an iterated integral with x and y as the variables of integration. Recalculate the integral using the change of variables $x = 2u + v$, $y = v$. *Obviously, the value of the integral shouldn't change.*

Extra Credit

Extra credit will be awarded for the best joke. All jokes must observe standards of good taste. The determination of the best joke will be made by popular vote in class when the papers are returned.