Mathematics 210 Name: ____________________
Professor Alan H. Stein
Due Monday, October 1, 2007 This problem set is worth 50 points.

Make sure that you check the course website for instructions, fill out the pledge form and hand it in with your paper. The instructions for problem sets and take-home examinations along with the pledge form are available from the *General Policies* portion of the web site. *No paper will be accepted without a signed pledge form.* Remember that your paper may be handed in before the deadline but that no late papers will be accepted regardless of the reason. The course website also includes an explanation of how your average will be calculated if you fail to complete this assignment.

Note that, since most of the calculations involved can be done routinely using either a calculator or a symbolic manipulation program such as Maple or Mathematica, it will obviously be necessary to show, through your work, exactly how you came up with your solutions.

Each part of each question will be given the same weight.

1. Find the angle between the vectors $(2, 7, 3)$ and $(5, -2, 4)$.
2. Find vector parametric, scalar parametric and scalar symmetric equations for the line through the point $(5, 3, 1)$ and orthogonal to the plane $3x - 7y + 2z = 8$.
3. Find an equation of the plane through the points $(1, 5, 4)$, $(4, 9, 1)$ and $(5, 17, 5)$. *Extra Credit: Obtain the same equation using an entirely di erent method.*
4. Find the distance between the point $(1, 2, 3)$ and the plane $4x + 5y + 6z = 7$. *Extra Credit: Find the distance using an entirely di erent method.*
5. An ant walks on the paraboloid $z = x^2+y^2$. Using the usual orientation for the coordinate axis, the ant starts at the origin and is rising at a rate of 4 units per second. Looking from above, it looks as if the ant is spiralling around the z-axis, starting by the x-axis and making a complete revolution every 5 seconds.
   (a) Find a vector parametric equation for the path of the ant.
   (b) Find the velocity of the ant.
   (c) Find the acceleration of the ant.

   Represent time by t. Each of your conclusions should be in terms of t.
6. Consider the vector function $\mathbf{x} = (t, 3 \sin t, 4 \cos t)$, $t \geq 0$.
   (a) Sketch the graph.
   (b) Find the length of the portion of the curve for which $\pi \leq t \leq 2\pi$. *Give your answer in terms of a definite integral. You do not need to evaluate the integral but may do so for extra credit.*

7. Consider the vector function $\mathbf{x} = (t, t^2, t^3)$, $t \geq 1$.
   (a) Sketch the graph.
   (b) Find the unit tangent vector $\mathbf{T}$ at $t = 2$.
   (c) Find the normal vector $\mathbf{N}$ at $t = 2$.
   (d) Find the binormal vector $\mathbf{B}$ at $t = 2$.
   (e) Verify $\mathbf{T}$, $\mathbf{N}$ and $\mathbf{B}$ are mutually orthogonal.
   (f) Find the curvature κ at $t = 2$.

8. A jogger runs leisurely at a constant pace of a six minute mile around a quarter mile track. The track consists of two parallel straighaways connected by two semicircles, each 110 yards long.
   (a) Set up an appropriate coordinate system.
   (b) Assuming the jogger starts at the beginning of one of the straightaways, represent the path of the jogger by a vector function. *Note: Your definition of the function will probably need at least four parts.*
   (c) Find the acceleration of the runner when he or she is exactly halfway through one of the semicircles. *Extra Credit: Obtain the acceleration again using an entirely di erent method.*