Sample Mathematics 116 Questions

(1) Let $f(x) = \int_2^{\ln x} \sqrt{1+t^2}\,dt$. Find $f'(x)$.
(2) Calculate $\int_2^5 w^2 - 4w + 3\,dw$.
(3) Calculate the following indefinite integrals.
  (a) $\int \dfrac{x}{1+x^2}\,dx$
  (b) $\int x(2x^2+3)\sqrt{x^4+3x^2+1}\,dx$
(4) $\mathcal{A}$ is the region bounded below by the $x-$axis, above by the line $y = 2x+5$, on the left by the vertical line $x = 2$ and on the right by the vertical line $x = 7$.

$\mathcal{B}$ is the region in the first quadrant bounded by the $y-$axis, the horizontal line $y = 2$ and the parabola $y^2 = x$.

$\mathcal{C}$ is the region inside the circle of radius 1 and center $(5, 0)$.
  (a) (i) Sketch each region
    (ii) Represent the area of each region in terms of definite integrals.
  (b) (i) Draw a rough sketch of each solid obtained by rotating each of the regions about the $x-$axis.
    (ii) Represent the volume of each region in terms of definite integrals.
  (c) (i) Draw a rough sketch of each solid obtained by rotating each of the regions about the $y-$axis.
    (ii) Represent the volume of each region in terms of definite integrals.
  (d) (i) Draw a rough sketch of each solid obtained by rotating each of the regions about the line $y = -2$.
    (ii) Represent the volume of each region in terms of definite integrals.
(5) Consider the region bounded by the graphs of the functions $f(x) = x+3$, $g(x) = 10x - x^2 - 15$.
  (a) Sketch the region.
  (b) Find its area.
  (c) Find the length of its boundary.
  (d) Find the volume of the solid of revolution obtained by rotating it about the $x-$axis.
  (e) Find the volume of the solid of revolution obtained by rotating it about the $y-$axis.
(6) For each of the following, determine whether or not the indicated limit exists. If it exists, calculate it.
  (a) $\lim_{x\to\infty} \dfrac{5x\ln x + 9x}{10x\ln x - 20x}$
  (b) $\lim \dfrac{5k\ln k + 9k}{10k\ln k - 20k}$
(7) (5 points) The mass of a certain radioactive element decreases from ten (10) grams to three (3) grams in two (2) hours. What is its half-life?
(8) Find the following indefinite integrals.
  (a) $\int \arctan x\,dx$

   (b) $\int \frac{x}{\sqrt{x^2+9}}\,dx$

(9) Consider the parametric equations $x = \cos t$, $y = t\sin t$, $0 \le t \le \pi$.
   (a) Sketch their graph.
   (b) Find the length of the curve your sketched.

(10) Consider the polar function $r = 5\sin(4\theta)$.
   (a) Sketch its graph.
   (b) Find its length.
   (c) Find the area of the region enclosed by its graph.

(11) Consider $\int_1^\infty \alpha^x\,dx$ for fixed $\alpha > 0$. For which values of α does the integral converge, and for which does it diverge? Justify your answer. For those values of α for which the integral converges, determine what it converges to.

(12) For each of the following series, determine whether it is absolutely convergent, conditionally convergent, or divergent. Justify your answer.
   (a) $\sum_{k=1}^\infty (-1)^k \frac{\ln k}{k}$
   (b) $\sum_{k=1}^\infty (-1)^k \frac{k}{1+3^k}$

(13) For each of the following functions, determine its Taylor Series, centered at 0, and the interval of convergence for its Taylor Series.
   (a) $f(x) = (x-1)^3$
   (b) $g(x) = \frac{1}{1-x}$

(14) Determine the interval of convergence for each of the following power series.
   (a) $\sum_{k=0}^\infty kx^k$
   (b) $\sum_{k=0}^\infty \frac{(x-3)^k}{4^k}$

(15) Assuming that it is known that if there is some constant β such that $0 \le a_k \le \beta b_k$ for all k and $\sum b_k$ converges then $\sum a_k$ converges, prove the following portion of the Limit Comparison Test: If $\lim \frac{a_k}{b_k} = \gamma$ for some positive constant γ and $\sum b_k$ converges, then $\sum a_k$ converges.

(16) Define the following.
   (a) The natural logarithm function
   (b) The exponential function
   (c) a^x
   (d) $\arcsin x$
   (e) $\cosh x$

(17) Calculate the derivatives of the following functions.
   (a) $f(x) = 10^x$
   (b) $g(t) = t/\arcsin(2t)$
   (c) $h(x) = \cosh^2 x$
   (d) $f(x) = e^{-x^2}$

(18) Calculate the following integrals.
   (a) $\int xe^{x^2}\,dx$

(b) $\int \frac{x}{1+x^4}\,dx$

(19) (a) A bank pays interest at an annual rate of 5%, compounded quarterly. How long will it take for the balance to double?

(b) Suppose there are 20 grams of a certain radioactive isotope that has a half life of 15 minutes. In how long will there be only 18 grams left?

(20) Calculate the following limits.

(a) $\lim_{x\to 2} \frac{x^2-x-2}{x^3-2x^2+3x-6}$

(b) $\lim_{x\to 0^+} \frac{\ln(1+x)}{x}$

(c) $\lim_{x\to\infty} x^2e^{-x}$

(d) $\lim_{x\to 0^+} x^x$

(21) Use logarithmic differentiation to show that if $f(x) = g(x)h(x)$ and $f'(x)$ exists, then $f'(x) = g(x)h'(x) + g'(x)h(x)$.

Consider each of the following integrals. If it is an indefinite integral, calculate it. If it is an improper integral, first determine whether or not it converges and then, if it converges, calculate its value.

(22) $\int \sin^4 x \cos^3 x\,dx$

(23) $\int \sin^4 x \cos^4 x\,dx$

(24) $\int e^{2x} \sin x\,dx$

(25) $\int x^2 \sin(2x)\,dx$

(26) $\int \sqrt{x^2-4}\,dx$

(27) $\int \frac{1}{\sqrt{x^2+4x+13}}\,dx$

(28) $\int \frac{x}{x^2-6x+9}\,dx$

(29) $\int_0^\infty x^2e^{-x}\,dx$

(30) $\int_1^\infty \frac{1}{\sqrt{x^2-1}}\,dx$

(31) $\int_1^\infty \frac{\ln x}{x^2}\,dx$

(32) Complete each of the following definitions.

(a) $\int_a^\infty f(x)\,dx$ converges if there is some $L \in \mathbb{R}$ such that for every $\epsilon > 0$ there is ....

(b) The sequence $\{a_k\}$ converges if there is some $L \in \mathbb{R}$ such that for every $\epsilon > 0$ there is ....

(c) The series $\sum_{k=1}^\infty a_k$ converges if there is some $L \in \mathbb{R}$ such that for every $\epsilon > 0$ there is ....

(33) For each of the following improper integrals, determine whether it is convergent or divergent, and clearly justify your answer. If it is convergent and it is feasible to determine what it converges to, do so.

(a) $\int_1^\infty \frac{1}{x\ln x}\,dx$

(b) $\int_2^4 \dfrac{1}{x^2-4}\,dx$
(c) $\int_4^\infty \dfrac{1}{x^2-4}\,dx$
(d) $\int_{-\infty}^\infty \dfrac{x}{e^{x^2}}\,dx$

(34) Consider the parametric equations

$$x = 3t\sin t \tag{1}$$
$$y = 5t\cos t \tag{2}$$
$$0 \le t \le 2\pi. \tag{3}$$

(a) Sketch their graph.
(b) Find the length of the graph in terms of a definite integral.
(c) If feasible, evaluate the integral.

(35) Consider the polar equation $r = 1 + 2\cos\theta$.
(a) Sketch its graph.
(b) Find the length of its graph in terms of a definite integral.
(c) If feasible, evaluate the integral.
(d) Find the area of the inside loop of its graph in terms of a definite integral.
(e) If feasible, evaluate the integral.

(36) For each sequence $\{a_k\}$, (i) write out the first three or four teerms and (ii) determine whether it is convergent or divergent, clearly justifying your answer. (iii) If it is convergent and it is feasible to determine what it converges to, do so.
(a) $a_k = (1)^k \dfrac{1}{k^2+1}$
(b) $a_k = \dfrac{8k^3-3k^2-7k+5}{4k^3+11k^2+5k-3}$
(c) $a_k = \dfrac{k}{k+1}$

(37) For each series $\sum_{k=4}^\infty a_k$, (i) write out the first three or four terms and (ii) determine whether it is convergent or divergent, clearly justifying your answer. (iii) If it is convergent and it is feasible to determine what it converges to, do so. *Note that the most important part is to justify your answer, and sometimes this may be even easier than you realize.*
(a) $a_k = (1)^k \dfrac{1}{k^2+1}$
(b) $a_k = \dfrac{8k^3-3k^2-7k+5}{4k^3+11k^2+5k-3}$
(c) $a_k = \dfrac{k}{k+1}$
(d) $a_k = (-1)^k \dfrac{k}{k+1}$
(e) $a_k = \dfrac{1}{5^k}$

(f) $a_k = \dfrac{k}{2^{k+1}}$
(g) $a_k = \dfrac{k}{1+k^4}$
(h) $a_k = \dfrac{1}{k \ln k}$
(i) $a_k = \dfrac{1}{k^2-4}$

(38) Evaluate the definite integral $\int_{\pi/6}^{\pi/3} (1+\sin^2 x)\cos x\, dx$.

Differentiate each of the following functions, putting your answer in an appropriate form.

(39) $f(x) = (x^2+1)^{\sin x}$
(40) $h(\theta) = e^{\ln(5\theta+3)}$
(41) Calculate exactly: $\int_{\pi/6}^{\pi/4} \dfrac{1}{\sqrt{1-x^2}}\, dx$
(42) Find: $\dfrac{d}{dx}\left(\dfrac{\arctan(3x-1)}{x^2}\right)$
(43) Find $\int \dfrac{\cos^2\theta}{\sin^3\theta}\, d\theta$.
(44) Find $\int \cot^2 x \csc x\, dx$
(45) Determine whether $\int_1^\infty \dfrac{1}{x^3}\, dx$ converges or diverges. If it converges, find its value.

Determine whether each series $\sum_{k=1}^{\infty}(-1)^k a_k$ converges absolutely, converges conditionally or diverges. Justify your answer.

(a) $a_k = \dfrac{k^2}{1+k^3}$.
(b) $a_k = \dfrac{k^2}{2^k}$

(46) According to Hooke's Law, the force of a spring is proportional to how far it is stretched past its equilibrium position. In other words, if the distance past its equilibrium position is represented by x and the force by $F(x)$, then $F(x) = kx$ for some constant k. Consider a spring which exerts a force of magnitude 10 when stretched a distance 2 past its equilibrium position.
(a) Determine the spring constant k.
(b) Determine the amount of work done in stretching the spring from its equilibrium position to 3 feet past its equilibrium position.

Note: You are now prepared to calculate how many calories you exert on certain exercise machines.

(47) Find the volume of the solid obtained by rotating the following plane region about the $x-$axis: The region bounded by the $x-$axis, the graph of $f(x) = \sqrt{x\sin(2x)}$, and the lines $x = 0$ and $x = \pi/2$.

(48) Without trying to calculate it, determine whether $\int_2^\infty \dfrac{1}{x^2-1}\, dx$ converges. If it converges, calculate it.

(49) Consider the parametric equations $x = 20t$, $y = 128 - 16t^2$, $t \geq 0$.
(a) Sketch the portion of the graph for which $y \geq 0$.
(b) Use a definite integral to represent the length of the graph you sketched.
(c) Calculate the length of the graph you sketched.
Note that you have calculated the length of the flight path of a ball tossed horizontally at an initial speed of 16 feet per second from a height 128 feet off the ground.

(50) (a) Sketch the graph of the polar equation $r = 4 - 3\sin\theta$.
(b) Use a definite integral to represent the length of the curve you sketched by a definite integral.
(c) Use a definite integral to represent the area of the region enclosed by the curve you sketched.
(d) Calculate the area of the region enclosed by the curve you sketched.

(51) Calculate $\lim_{x\to\infty} x \ln\left(1 + \frac{1}{x}\right)$.

(52) Consider a sequence $\{a_n\}$ defined recursively by $a_1 = 0$, $a_{n+1} = \dfrac{1 + a_n}{2}$.
(a) Prove: If $a_n < 1$, then $a_{n+1} < 1$.
(b) Write down a simple formula for a_n.
(c) Calculate $\lim a_n$.
Hint: Don't be intimidated by the recursive definition. If you write out the first few terms, everything should fall into place.

(53) Determine whether the series $\sum_{n=0}^{\infty} (-1)^n \frac{n}{3^n}$ converges absolutely, converges conditionally, or diverges.

(54) Consider the power series $p(x) = 1 - x^2 + x^4 - x^6 \pm \dots$.
(a) Find its radius of convergence.
(b) Find its interval of convergence.
(c) Obtain a simple formula for the function $f(x)$ which $p(x)$ converges to within its interval of convergence.
(d) Find $\int f(x)\,dx$.

(55) (a) Find the Taylor Series centered at 0 for $f(x) = \sinh(x)$.
(b) Find its interval of convergence.

(56) Assuming that it is known that if there is some constant β such that $0 \leq a_k \leq \beta b_k$ for all k and $\sum b_k$ converges then $\sum a_k$ converges, prove the following portion of the Limit Comparison Test: If $\lim \dfrac{a_k}{b_k} = \gamma$ for some positive constant γ and $\sum b_k$ converges, then $\sum a_k$ converges.

(57) (10 points) Consider the natural logarithm function ln.
(a) Define $\ln(x)$.) conlosequ6(to)-325(repr26(?137.90250 Td947 Td[((c))e416(Find)]Tt:)-o(equa

(g) Analyze its monotonicity.
(h) Analyze its concavity.
(i) Sketch its graph.

(58) Consider the plane region bounded by two coordinate axes, the line $x = 2$ and the curve $y = x^2 - 2x + 2$.
(a) Sketch the region and find its area.
(b) Sketch the solid obtained by rotating the region about the $x-$axis and find its volume. *You may leave the volume in terms of a definite integral.*
(c) Sketch the solid obtained by rotating the region about the $y-$axis and find its volume. *You may leave the volume in terms of a definite integral.*

(59) Calculate the derivatives of the following functions.
(a) $f(x) = x^2 e^x$
(b) $g(t) = (t^2 + 1)^{5t}$
(c) The function $y = f(x)$ defined implicitly by the equation $x = \ln(y)$.

(60) Calculate the following integrals.
(a) $\int \frac{\cos x}{\sin x}\, dx$
(b) $\int_1^{10} \frac{1}{x}\, dx$
(c) $\int (2x + 1)e^{x^2+x+1}\, dx$

(61) It is determined that a specimen from a fossil contains 2.3 grams of carbon-14 and would have contained 0.4 grams of carbon-14 while it was alive. Assuming that the half-life of carbon-14 is 5750 years, how old is the fossil?

(62) Consider the function $f(x) = e^{x^2-4x}$.
(a) Calculate f' and f''.
(b) Analyze its monotonicity.
(c) Analyze its concavity.
(d) Find and clearly identify all extrema.
(e) Find and clearly identify all points of inflection.
(f) Sketch its graph.

(63) $\lim_{x\to 25} \frac{x^2 - 625}{\sqrt{x} - 5}$

(64) $\lim_{x\to\infty} \frac{x^8 + 3x^2 - 2x + 1}{2^x}$

(65) $\lim_{\theta\to\infty} \frac{\sin\theta}{\theta}$

(66) $\lim_{t\to\infty} (\ln t)^{1/t}$

(67) $\lim_{z\to 1^+} (z - 1)^{\ln z}$

(68) $\int \sin^4 x \cos^3 x\, dx$

(69) $\int \sin^4 x \cos^4 x\, dx$

(70) $\int e^{2x} \sin x\, dx$

(71) $\int x^2 \sin(2x)\, dx$

(72) $\int \sqrt{x^2 - 4}\, dx$

(73) $\int \frac{1}{\sqrt{x^2+4x+13}}\,dx$

(74) $\int \frac{x}{x^2-6x+9}\,dx$

(75) $\int x^2 e^{-x}\,dx$

(76) $\int \frac{1}{\sqrt{x^2-1}}\,dx$

(77) $\int x^8 \ln x\,dx$

(78) The weight of an object is the force exerted on it by the earth's gravity. The force of gravity follows the inverse square law, meaning that and object's weight $F(x)$ is related to its distance x from the center of the earth by the formula $F(x) = k/x^2$ for some constant k. Consider a satellite of weight 10,000 pounds on the earth's surface and assume that the earth is a perfect sphere of radius 4000 miles.
   (a) Find the constant k for the satellite.
   (b) Find the amount of work which must be performed to lift the satellite to an orbital position 1000 miles above the earth's surface.
   (c) Find the amount of work which must be performed to lift the satellite free of the earth's gravitational field.

(79) Prove that $\int_0^\infty e^{px}\,dx$ converges for $p < 0$ and diverges for $p \geq 0$.

(80) Determine whether $\int_0^4 \frac{x}{\sqrt{16-x^2}}\,dx$ converges or diverges. Justify your conclusion. If it converges, evaluate it.

(81) Consider the curve given by the parametric equations $x = 3\sin t$, $y = 2\cos t$, $0 \leq t \leq 3\pi/2$.
   (a) Sketch its graph. Clearly indicate points on the graph corresponding to key values of t.
   (b) Represent the length of the graph as a definite integral.

(82) Consider the curve given in polar coordinates by the equation $r = 5\theta$, $-2\pi \leq \theta \leq 2\pi$.
   (a) Sketch its graph.
   (b) Find its length.

(83) Consider the curve given in polar coordinates by the equation $r = 3 + 4\cos\theta$.
   (a) Sketch its graph.
   (b) Find the area of the region it encloses.

(84) Consider the sequence $\{a_n\}$ whose first few terms are $\frac{1}{3}, \frac{3}{5}, \frac{5}{7}, \ldots$.
   (a) Find a formula for a_n.
   (b) Find $\lim a_n$.

(85) Evaluate $\lim \frac{3n^2 - 2n\ln n + 8}{2n^2 + n + 1}$.

For each infinite series, determine whether or not it converges. If it converges, determine whether it converges absolutely or conditionally. Justify your answer.

(86) $\sum_{k=1}^\infty (-1)^k \frac{1}{k(k+1)}$

(87) $\sum_{k=1}^{\infty}(-1)^k \dfrac{2k}{2k+1}$

(88) Consider the series $\sum_{k=1}^{\infty} \dfrac{1}{k} - \dfrac{1}{k+3}$.

(a) Find a formula in closed form for the nth partial sum s_n. *In plain language, find a simple formula for* s_n.

(b) Determine the sum of the series.

(89) A ball is dropped from a height of 15 feet. Each time it strikes the ground it bounces vertically to a height that is 80% of the height it reached on its previous bounce. Find the total distance the ball will travel if it is allowed to bounce indefinitely.

For each power series, determine its interval of convergence. *Don't neglect the endpoints.* Justify your answer.

(90) $\sum_{k=0}^{\infty} 3^k x^k / k^2$

(91) $\sum_{k=0}^{\infty} x^k / \sqrt{k}$

(92) Find the Taylor Series centered at 0 for $\sqrt{1+x}$.

(93) Calculate $\int_0^{\pi} \sin^2 x \cos x \, dx$.

(94) Sketch the region bounded by the graphs of $f(x) = 7x - 4$ and $g(x) = x^2 + 3x - 1$ and find its area.

Consider the plane region $\mathcal{R}$ bounded by the $x-$axis, the line $x = 3$, the line $x = 8$ and the graph of the function $f(x) = x^2 - 10 + 28$.

(95) Find the volume of the solid obtained by rotating the region $\mathcal{R}$ about the $x-$axis.

(96) Find the volume of the solid obtained by rotating the region $\mathcal{R}$ about the $y-$axis.

(97) Find formulas for the derivatives of each of the following functions.

(a) $f(x) = (x^2 + 3x - 1)^{x^2}$ *Extra Credit: What is the domain of f'?*

(b) $g(t) = \ln\left(\dfrac{t^4}{1+t^2}\right)$

(98) Calculate the following integrals.

(a) $\int_1^5 \dfrac{1}{x} \, dx$

(b) $\int x \exp\left(x^2\right) \, dx$

(99) Consider the function $f(x) : [0, \pi] \to [-1, 1]$ defined by the formula $f(x) = \cos x$.

(a) Find a formula for derivative of $f(x)$.

(b) Find a formula for the derivative of the inverse of $f(x)$.

(100) Suppose \$450 is placed in a bank account paying interest at an annual rate of 3.5%, compounded continuously. What will the balance be after two years?

(101) Find $\arcsin(-1/2)$.

(102) Find $\dfrac{d}{dx}\left(x \arctan(x)\right)$.

(103) Find $\int \dfrac{1}{1+x^2} \, dx$.

(104) Find $\int x^4 \ln x \, dx$.

(105) Find $\int \sin^3 x \cos^2 x \, dx$.

(106) Find $\dfrac{d}{dx}\left(\int x^3 (\arctan x) e^{3x-1} \, dx\right)$.

For each of the following improper integrals, determine whether it is convergent or divergent. If it is convergent and it is feasible to determine what it converges to, do so.

(107) $\int_2^\infty \dfrac{1}{x^2+4}\,dx$

(108) $\int_0^1 \dfrac{1}{x^2}\,dx$

(109) Consider the parametric equations

$$x = \sin t$$
$$y = \sin^2 t$$
$$0 \le t \le 2\pi.$$

(a) Sketch the graph.

(b) Represent the length of the curve you sketched as a definite integral. Simplify the integrand as much as you can.

(110) Consider the plane region bounded by the $x-$axis and the curve $y = 1 - x^2$.

(a) Sketch the region.

(b) Represent the area of the region by a definite integral.

(c) Sketch the solid obtained by rotating the region about the $y-$axis.

(d) Represent the volume of the solid by a definite integral where the shell method is used.

(e) Represent the volume of the solid by a definite integral where the disk method is used.

(111) Determine whether the sequence $\{a_n\}$, where $a_n = n\ln\left(1+\frac{3}{2n}\right)$, converges. If it converges, find its limit.

(112) Find $\int xe^{2x}\,dx$.

(113) Find $\int \cos^3 x\,dx$.

(114) Evaluate $\int_1^\infty \dfrac{1}{x^4}\,dx$.

(115) Determine whether the series $\sum_{n=1}^\infty (-1)^n \dfrac{\ln n}{n^3}$ converges absolutely, converges conditionally or diverges.

(116) Consider the power series $\sum_{n=0}^\infty (-1)^{n+1} x^n = 1 - x + x^2 - x^3 + \cdots$.

(a) Find its radius of convergence.

(b) Find its interval of convergence.

(c) Prove that it converges to $\frac{1}{1+x}$ within its interval of convergence.

(d) Integrate the series term by term to obtain a power series that converges to $\ln(1+x)$ for $x \in (-1, 1]$.

(e) User your answer to the last part to obtain an ordinary infinite series which is conditionally convergent and converges to $\ln 2$.

(117) Consider the Taylor Series $\sum_{n=0}^\infty a_n x^n$, centered at 0, for e^{2x}. Find a formula for a_n and write out the first five terms of the series.

(118) Consider the plane region $\mathcal{R}$ bounded by the $x-$axis, the lines $x = 2$ and $x = 4$ and the curve $y = x^2e^x$. Represent each of the following by definite integrals. *Do not carry out the integration.*

- (a) The area of $\mathcal{R}$.
- (b) The volume of the solid obtained by rotating the region $\mathcal{R}$ about the $x-$axis.
- (c) The volume of the solid obtained by rotating the region $\mathcal{R}$ about the $y-$axis.

(119) (25 points) Find the derivatives of each of the following functions.

- (a) $f(x) = \ln(e^{5x})$
- (b) $g(t) = \dfrac{t}{\arctan t}$
- (c) The function $y = f(x)$ defined implicitly by the equation $x = e^{3y}$. *Extra Credit: Determine the domain and range of f.*

(120) Evaluate each of the following exactly, without the use of artifical aids such as calculators.

- (a) $\sin(\arccos .3)$
- (b) $\arccos(-1/2)$
- (c) $\arcsin(\sin(7\pi/6))$

(121) Find $\int \dfrac{1}{x^2+4}\,dx$.

(122) Find $\int \dfrac{x}{x^2+4}\,dx$.

(123) Find $\int xe^{2x}\,dx$.

(124) Suppose a bacteria population starts with 100 bacteria and doubles every three hours. When will the population reach 100,000? *Find both an exact answer, ignoring the fact that population is a discrete, non-continuous and non-differentiable function, and then give a decimal approximation.*

(125) Assuming that the half-life of Strontium-90 is 25 years, how long will it take before an initial mass of 24 milligrams is reduced to 5 milligrams?

(126) Consider the plane region A bounded by the x-axis and the graph of the function $f(x) = 4 - (x - 10)^2$, the solid B obtained by rotating the plane region A about the x-axis and the solid C obtained by rotating the plane region A about the y-axis

- (a) Sketch the region A.
- (b) Set up a definite integral the value of which is the area of the region A.
- (c) Sketch the solid B.
- (d) Set up a definite integral the value of which is the volume of the solid B.
- (e) Sketch the solid C.
- (f) Set up a definite integral the value of which is the volume of the solid C.

(127) Consider the function $f(x) = x \ln x$.

- (a) What is the domain of f?
- (b) Find $f'(x)$ and $f''(x)$.
- (c) Determine where f' is positive and where f' is negative.
- (d) Sketch the graph of f.

(128) Consider the function $f(x) = (x^2 + 1)^{3x}$. Find $f'(x)$ two different ways.

(129) Let $f(x) = \tan(\arcsin x)$.

(a) What is the domain of f?
(b) Find $f'(x)$ without first simplifying $f(x)$.
(c) Simplify $f(x)$, writing it as an algebraic function.
(d) Use the above to find $f'(x)$.

(130) $\int (\ln x)^2\, dx$
(131) $\int x^3\sqrt{16-x^2}\, dx$
(132) Calculate $\lim_{x\to 0} \frac{\sin x - x}{x^3}$.
(133) Calculate $\lim_{x\to\infty} \left(\frac{x+1}{x}\right)^x$.
(134) Each part of this question refers to the curve $y = x^2\ln(3x+2)$ between $x = 2$ and $x = 4$. For each part of this question, draw a rough but relevant sketch and set up a definite integral whose value represents the desired quantity. *Do not evaluate any of the integrals.*
(a) The length of the curve.
(b) The surface area of the surface obtained by rotating the curve about the $x-$axis.
(c) The surface area of the surface obtained by rotating the curve about the $y-$axis.

(135) Each part of this question refers to plane region bounded by the graph of the curve $y = xe^x$, the $x-$axis and the lines $x = 3$ and $x = 5$. For each part of this question, draw a rough but relevant sketch and set up a definite integral whose value represents the desired quantity. *Do not evaluate any of the integrals.*
(a) The area of the plane region.
(b) The volume of the solid obtained by rotating the plane region about the $x-$axis.
(c) The volume of the solid obtained by rotating the plane region about the $y-$axis.

(136) A force of 15 pounds is needed to stretch a spring 3 feet beyond its equilibrium position. How much work must be done to stretch the spring that far. *Obviously, this is a big spring.*
(137) Let $f(x) = \dfrac{\sin x}{e^{3x}}$. Find $f'(x)$.
(138) Let $f(x) = (\sin x)^{\cos x}$. Find $f'(x)$. *Extra Credit: Find $f'(x)$ two different ways.*
(139) Find $\int x^2e^x\, dx$. *Extra Credit: Find $\int x^2e^x\, dx$ two different ways.*
(140) Find $\int \dfrac{\cos^3 x}{1-\cos^2 x}\, dx$. *Hint:* $\cos^x + \sin^2 x = 1$.
(141) Find $\int \dfrac{1}{u^2(1+u^2)^{3/2}}\, dx$.
(142) Find $\lim_{x\to 0} \dfrac{\sin^2(3x)}{1-\cos x}$. *Extra Credit: Find the limit two different ways.*
(143) Consider the region bounded by the curves $y = x^2$ and $y = 8 - x^2$.
(a) Sketch the region.
(b) Find the area of the region.
(c) Consider the solid obtained rotating the region about the $x-$axis. Represent the volume of the solid by a definite integral. *Do not evaluate the integral.*
(d) Consider the solid obtained rotating the region about the $y-$axis. Represent the volume of the solid by a definite integral. *Do not evaluate the integral.*

(144) Evaluate $\int x^2 \sin x\, dx$.

(145) Evaluate $\int \dfrac{1}{\sqrt{x^2+4}}\,dx$.

(146) Consider the parametric curve $x = 2\cos t$, $y = \sin^2 t$, $0 \le t \le \pi$.
   (a) Sketch the curve, indicating the direction by an arrow.
   (b) Find the slope of the tangent at the point $t = 0$.
   (c) Find the length of the curve. *Represent the length by a definite integral, but do not evaluate the integral.*
   (d) Find the area of the region between the curve and the $x-$axis. *Represent the area by a definite integral, but do not evaluate the integral.*
   (e) (Extra Credit) Eliminate the parameter t and represent the curve by an ordinary equation.

(147) Consider the polar curve $r = \sin 4\theta$.
   (a) Sketch the curve.
   (b) Find the length of the curve. *Represent the length by a definite integral, but do not evaluate the integral.*
   (c) Find the area of the region bounded by the curve. *Represent the area by a definite integral, but do not evaluate the integral.*

(148) Consider the vectors $\mathbf{u} =< 3, 8 >$, $\mathbf{v} =< 1, 13 >$.
   (a) Find $\mathbf{u} + \mathbf{v}$.
   (b) Find $\mathbf{u} - \mathbf{v}$.
   (c) Find $5\mathbf{u}$.
   (d) Find $\mathbf{u} \cdot \mathbf{v}$.

(149) Define what is meant by the **limit of a sequence.** *Hint: Recall the definition of a limit of an ordinary function at infinity,* $\lim_{x\to\infty} f(x) = L$ *if for every positive number* $\epsilon > 0$ *there is some real number* M *such that* $|f(x) - L| < \epsilon$ *whenever* $x > M$.

(150) (a) Find $\lim_{x\to\infty} x^{1/x}$.
   (b) Find $\lim \sqrt[n]{n}$.

(151) Consider the series $\sum_{n=1}^{\infty} \dfrac{1}{n^2}$. We know that if the limit of the series is estimated by the n^{th} partial sum s_n then the error E_n is no greater than $\int_n^{\infty} \dfrac{1}{x^2}\,dx$.
   (a) Find a bound for E_n in terms of n.
   (b) How large must n be in order to guarantee that $E_n \le 10^{-10}$?
   (c) (Extra Credit) Prove that $E_n \le \int_n^{\infty} \dfrac{1}{x^2}\,dx$.

(152) (a) Determine whether the improper integral $\int_1^{\infty} \dfrac{\sqrt{x}}{x+4}\,dx$ converges.
   (b) Determine whether the series $\sum_{n=1}^{\infty} (-1)^n \cdot \dfrac{\sqrt{n}}{n+4}$ converges absolutely, converges conditionally or diverges.

(153) (a) Determine whether the improper integral $\int_0^{\infty} \dfrac{x}{e^x}\,dx$ converges.
   (b) Determine whether the series $\sum_{n=0}^{\infty} (-1)^n \cdot \dfrac{n}{2^n}$ converges absolutely, converges conditionally or diverges.

(154) Find the radius of convergence and the interval of convergence of the power series $\sum_{n=1}^{\infty} \dfrac{x^n}{n \cdot 10^n}$.

(155) Sketch the plane region bounded by the curve $y = x^2 - 6x + 20$, the $x-$axis and the lines $x = 3$ and $x = 5$ and represent the area of the region in terms of a definite integral.

(156) Sketch the solid obtained by rotating that plane region about the $x-$axis and represent the volume of the solid obtained in terms of a definite integral.

(157) Sketch the solid obtained by rotating that plane region about the $y-$axis and represent the volume of the solid obtained in terms of a definite integral.

(158) Each of the following questions refer to the portion for $2 \leq x \leq 5$ of the graph of the function $f(x) = x + \ln x$. For each question, set up a definite integral whose value is equal to the quantity asked for but do not calculate the integral.

(a) Find the length of the curve.

(b) Find the area of the plane region below the curve but above the $x-$axis.

(c) Find the volume of the solid obtained by rotating the plane region about the $x-$axis.

(d) Find the volume of the solid obtained by rotating the plane region about the $y-$axis.

(e) Find the volume of the solid obtained by rotating the plane region about the line $x = 1$.

(f) Find the surface area of the surface obtained by rotating the curve about the $x-$axis.

(g) Find the surface area of the surface obtained by rotating the curve about the $y-$axis.

(h) Find the surface area of the surface obtained by rotating the curve about the line $x = 1$.