Mathematics 116 Name: _____

Professor Alan H. Stein

Due Friday, March 8 This problem set will be graded on the basis of 100 points but will be worth 50 points.

Make sure that you check the course website for instructions, fill out the pledge form and hand it in with your paper. The instructions for problem sets and take-home examinations along with the pledge form are available from the *General Policies* portion of the web site. *No paper will be accepted without a signed pledge form.* Remember that your paper may be handed in before the deadline but that no late papers will be accepted regardless of the reason. The course website also includes an explanation of how your average will be calculated if you fail to complete this assignment.

Note that, since most of the calculations involved can be done routinely using either a calculator or a symbolic manipulation program such as Maple or Mathematica, it will obviously be necessary to show, through your work, exactly how you came up with your solutions.

1. Calculate each of the following derivatives.
   (a) $\dfrac{d}{dx}\left(\sin(\ln x)\right)$.
   (b) $\dfrac{d}{dx}\left(\cos(\ln x)\right)$.
   (c) $\dfrac{d}{dx}\left(\tan(\ln x)\right)$.
   (d) $\dfrac{d}{dx}\left(\sec(\ln x)\right)$.
   (e) $\dfrac{d}{dx}\left(\csc(\ln x)\right)$.
   (f) $\dfrac{d}{dx}\left(\cot(\ln x)\right)$.
   (g) $\dfrac{d}{dx}\left(\arcsin(\ln x)\right)$.
   (h) $\dfrac{d}{dx}\left(\arccos(\ln x)\right)$.
   (i) $\dfrac{d}{dx}\left(\arctan(\ln x)\right)$.
   (j) $\dfrac{d}{dx}\left(\ln(\ln x)\right)$.
   (k) $\dfrac{d}{dx}\left(\exp(\ln x)\right)$.
   (l) $\dfrac{d}{dx}\left(\int_5^{\ln x}(t^3+17)\arctan t\,dt\right)$.

2. A population of rabbits grows exactly 5 percent per year. How long does it take for the population to triple? *Ignore the reality that the population is a discrete function of time and use an exponential model to obtain a reasonable answer.*

3. A fossil contains 3 percent of the amount of a radioactive substance it had when it was part of a living organism. Assuming the radioactive substance has a half-life of 247 years, find the age of the fossil. *Your solution should be self-contained, assuming only that the amount of a radioactive substance is an exponential function of time.*

4. Define the arctan function and use that definition to show $\dfrac{d}{dx}(\arctan x)$ must be $\dfrac{1}{1+x^2}$.

(5-10: Calculate the following limits. Extra credit will be given for any computation correctly done without the use of L'Hôpital's Rule.

5. $\lim_{x\to 0} \dfrac{\sin 6x}{\sin 2x}$

6. $\lim_{x\to\infty} \dfrac{(\ln x)^2}{x}$

7. $\lim_{x\to 0^+} (\sin x) \ln x$

8. $\lim_{x\to\infty} \left(\dfrac{2x+3}{2x}\right)^x$

9. $\lim_{x\to 0} \sin(8x)\cot(2x)$

10. $\lim_{x\to\infty} (\sqrt{x^2+4x} - x)$

Extra Credit

Extra credit will be awarded for the best joke. All jokes must observe standards of good taste. The determination of the best joke will be made by popular vote in class when the papers are returned.