Mathematics 115 Name: _____
Professor Alan H. Stein
Due Friday, April 20, 2007 This problem set is worth 50 points.

Make sure that you check the course website for instructions, fill out the pledge form and hand it in with your paper. The instructions for problem sets and take-home examinations along with the pledge form are available from the *General Policies* portion of the web site. *No paper will be accepted without a signed pledge form.* Remember that your paper may be handed in before the deadline but that no late papers will be accepted regardless of the reason. The course website also includes an explanation of how your average will be calculated if you fail to complete this assignment.

Note that, since most of the calculations involved can be done routinely using either a calculator or a symbolic manipulation program such as Maple or Mathematica, it will obviously be necessary to show, through your work, exactly how you came up with your solutions.

1. (a) Define relative minimum.
   (b) Define strictly increasing.
   (c) Using plain language and avoiding all use of mathematical notation, write down a strategy for sketching graphs.
   (d) Using plain language and avoiding all use of mathematical notation, write down a strategy for tackling optimization problems.

2. A function f is continuous and differentiable everywhere except where the information given implies otherwise. It is strictly increasing on $(-\infty, -3) \cup (-3, 4) \cup (8, \infty)$ and strictly decreasing on $(4, 8)$. It is concave up on $(-\infty, -3) \cup (6, \infty)$ and concave down on $(-3, 6)$. $\lim_{x \to -\infty} f(x) = 5$, $\lim_{x \to -3^-} f(x) = \infty$, $\lim_{x \to -3^+} f(x) = -\infty$, $f(0) = -3$, $f(4) = 7$, $f(6) = 5$, $f(8) = 2$. Sketch its graph and completely identify all relative and absolute extrema, all points of inflection, all discontinuities and all asymptotes.

3. Let $f(x) = x^3 - 12x$. Sketch its graph and completely identify all relative and absolute extrema, all points of inflection, all discontinuities and all asymptotes.

4. Let $f(x) = \sin x + \cos x$. Sketch its graph and completely identify all relative and absolute extrema, all points of inflection, all discontinuities and all asymptotes. *Extra Credit: Show how the graph can be sketched without using any Calculus.*

5. An automobile is traveling at a speed of 80 feet per second when the driver sees a young child in the road 195 feet in front. It takes a half second before he steps on the brakes as fast as he can, at which point the car slows down 20 feet per second each second until it stops. Does the car hit the child? *Use only the tools of Calculus; do not use any formulas you may have memorized from other courses such as physics.*

6. Find the point on the hyperbola $xy = 1$ which is closest to the origin. *The conclusion may seem obvious, but it must be justified.*

7. An open rectangular box has a square base and a capacity of 250 cubic inches. The material used for the bottom costs one cent per square inch, while the material used for the sides only costs half a cent per square inch. The box was designed so the cost for the material would be as little as possible. How much did the material cost?

8. Calculate $\int \sin^4\theta \cos\theta \, d\theta$.

9. Calculate $\displaystyle\int_{\pi/6}^{\pi/3} \sin x \, dx$.

10. Let $f(x) = \displaystyle\int_5^{x^2} \frac{\sin t}{t^4+9} \, dt$. Find $f'(x)$.

Extra Credit

Extra credit will be awarded for the best joke. All jokes must observe standards of good taste. The determination of the best joke will be made by popular vote in class when the papers are returned.