Mathematics 115 SOLUTIONS
Professor Alan H. Stein
Due Friday, February 9, 2007 This problem set is worth 50 points.

(1-8) Let $f(x) = \begin{cases} x^2 & \text{for } x < 0 \\ 1 & \text{for } x = 0 \\ \dfrac{x^2+x-42}{x-6} & \text{for } 0 < x < 10 \\ \dfrac{5x^2+3x-1}{x^2-144} & \text{for } x > 10. \end{cases}$

Find the limits indicated. If a limit doesn't exist, show why.

1. $\lim_{x\to -5} f(x)$

   Solution: $\lim_{x\to -5} f(x) = (-5)^2 = 25$

2. $\lim_{x\to 0} f(x)$

   Solution: This limit doesn't exist, since the left and right hand limits, calculated in the next two questions, are different.

3. $\lim_{x\to 0^-} f(x)$

   Solution: $\lim_{x\to 0^-} f(x) = \lim_{x\to 0^-} x^2 = 0$

4. $\lim_{x\to 0^+} f(x)$

   Solution: $\lim_{x\to 0^+} f(x) = \lim_{x\to 0^+} \dfrac{x^2+x-42}{x-6} = 7$

5. $\lim_{x\to 6} f(x)$

   Solution: $\lim_{x\to 6} f(x) = \lim_{x\to 6} \dfrac{x^2+x-42}{x-6} = \lim_{x\to 6} \dfrac{(x-6)(x+7)}{x-6}$
   $= \lim_{x\to 6}(x+7) = 13.$

6. $\lim_{x\to 12^-} f(x)$

   Solution: $\lim_{x\to 12^-} f(x) = \lim_{x\to 12^-} \dfrac{5x^2+3x-1}{x^2-144} = -\infty$, since the numerator is close to some positive number when $x \approx 12$ while the denominator is close to 0 but negative when x is a little less than 12.

7. $\lim_{x\to 12^+} f(x)$

   Solution: $\lim_{x\to 12^+} f(x) = \lim_{x\to 12^+} \dfrac{5x^2+3x-1}{x^2-144} = \infty$, since the numerator is close to some positive number when $x \approx 12$ while the denominator is close to 0 but positive when x is a little bigger than 12.

8. $\lim_{x\to\infty} f(x)$

   Solution: $\lim_{x\to\infty} f(x) = \lim_{x\to\infty} \dfrac{5x^2+3x-1}{x^2-144} = \lim_{x\to\infty} \dfrac{x^2(5+3/x-1/x^2)}{x^2(1-144/x^2)} =$
   $\lim_{x\to\infty} \dfrac{5+3/x-1/x^2}{1-144/x^2} = 5$

9. Find $\lim_{x\to 8} \dfrac{\frac{1}{x+2}-\frac{1}{10}}{x-8}$.

   Solution: $\lim_{x\to 8} \dfrac{\frac{1}{x+2}-\frac{1}{10}}{x-8} = \lim_{x\to 8} \dfrac{\frac{1}{x+2}-\frac{1}{10}}{x-8} \cdot \dfrac{10(x+2)}{10(x+2)} =$
   $\lim_{x\to 8} \dfrac{10-(x+2)}{(x-8)\cdot 10(x+2)} = \lim_{x\to 8} \dfrac{8-x}{(x-8)\cdot 10(x+2)} = \lim_{x\to 8} \dfrac{-1}{10(x+2)} = -\dfrac{1}{100}$

10. Find $\lim_{x\to 5} \dfrac{\sqrt{x+4}-3}{x-5}$.

    Solution: $\lim_{x\to 5} \dfrac{\sqrt{x+4}-3}{x-5} = \lim_{x\to 5} \dfrac{\sqrt{x+4}-3}{x-5} \cdot \dfrac{\sqrt{x+4}+3}{\sqrt{x+4}+3} =$
    $\lim_{x\to 5} \dfrac{x-5}{(x-5)(\sqrt{x+4}+3)} = \lim_{x\to 5} \dfrac{1}{\sqrt{x+4}+3} = \lim_{x\to 5} \dfrac{1}{6}$

11. Find $\lim_{x\to 5} \dfrac{|x-10|-5}{x-5}$.

    Solution: $\lim_{x\to 5} \dfrac{|x-10|-5}{x-5} = \lim_{x\to 5} \dfrac{(10-x)-5}{x-5} = \lim_{x\to 5} \dfrac{5-x}{x-5} = \lim_{x\to 5}(-1) =$
    -1

12. Find some $\delta > 0$ such that $|f(x) - 19| < .01$ when $|x - 3| < \delta$, where $f(x) = 4x + 7$.

    Solution: Each of the following inequalities is equivalent to each of the others:
    $|f(x) - 19| < .01$
    $|(4x + 7) - 19| < .01$
    $|4x - 12| < .01$
    $|4(x - 3)| < .01$
    $4|x - 3| < .01$
    $|x - 3| < \frac{.01}{4}$

    We may thus take $\delta = \frac{.01}{4}$.

13. Use the Bisection Method, starting with the interval $[0, 1]$, to find an interval of width no greater than 0.1 which contains a zero of the polynomial $x^5 + x^2 - 1$.

    Solution: Letting $f(x) = x^5 + x^2 - 1$, the following matrix shows the values of $f(x)$ at the endpoints of each interval along with the value at the midpoint. Each interval