Properties of Limits

When calculating limits, we intuitively make use of some basic properties it's worth noting. Each can be proven using a formal definition of a limit.

We list some of them, usually both using mathematical notation and using plain language. It is the plain language that should be remembered. In each case, unless noted otherwise, we assume the limits written down actually exist. As is usually the case, x, y, z, u, v, s, t will represent variables and a, b, c, k, L will represent constants.

Properties of Limits

- $\lim_{x\to c} k = k$ The limit of a constant is that constant.
- $\lim_{x\to c} x = c$ When x gets close to c, x gets close to c.
- $\lim_{x\to c}[kf(x)] = k\lim_{x\to c} f(x)$ The limit of a constant times a function is equal to the constant times the limit.
- $\lim_{x\to c}[f(x) \pm g(x)] = \lim_{x\to c} f(x) \pm \lim_{x\to c} g(x)$ The limit of a sum or di�erence is the sum or di�erence of the limits.
- $\lim_{x\to c}[f(x) \cdot g(x)] = [\lim_{x\to c} f(x)] \cdot [\lim_{x\to c} g(x)]$ The limit of a product is the product of the limits.
- $\lim_{x\to c} \dfrac{f(x)}{g(x)} = \dfrac{\lim_{x\to c} f(x)}{\lim_{x\to c} g(x)}$ The limit of a quotient is equal to the quotient of the limits. *Of course, this depends on the quotient existing, so the denominator on the right must not equal 0.*

Limit of a Composite Function

$\lim_{x\to c} f \circ g(x) = \lim_{x\to c} f(g(x))$
$= f(\lim_{x\to c} g(x))$ if f is continuous at
$\lim_{x\to c} g(x)$.

The limit of a composition is the composition of the limits, provided the outside function is continuous at the limit of the inside function.

Example: $\lim_{x\to 3} \sqrt{5x+1} = \sqrt{16} = 4$.

The Squeeze Theorem

Theorem 1 (Squeeze Theorem). *If $f(x) \le g(x) \le h(x)$ in some open interval containing c and*
$\lim_{x\to c} f(x) = \lim_{x\to c} h(x) = L$, then
$\lim_{x\to c} g(x) = L$.

If the values of a function lie between the values of two functions which have the same limit, then that function must share that limit.