

Your signature is your pledge that you have adhered to the guidelines for problem sets and take-home examinations.

This problem set will be graded on the basis of 100 points but will be worth 50 points.

Make sure that you check the course website for instructions. Remember that your paper may be handed in before the deadline but that no late papers will be accepted regardless of the reason.

This problem set is *optional* for those students who have not missed any other exams or problem sets. If it is submitted by the due date, it will be graded and used to compute your average. If you have submitted the other exams and problem sets but this is not submitted, your average will be calculated based on your other grades and there will be no penalty for not submitting this problem set.

Note that, since most of the calculations involved can be done routinely using either a calculator or a symbolic manipulation program such as Maple or Mathematica, it will obviously be necessary to show, through your work, exactly how you came up with your solutions.

1. Find a bound  $B$  such that  $|f''(x)| \leq B$  for  $-2 \leq x \leq 3$ , where  $f(x) = x^2 e^{-x} \sin x$ . *Make sure each step you take is clear. Do not use any information about the size of  $e$  other than  $2 < x < 3$  and do not use a calculator more than once.*
2. Assume you need to use Simpson's Rule to estimate  $\int_{-2}^3 f(x) dx$  with an error no greater than  $5 \cdot 10^{-12}$  and you have found that  $|f^{(4)}(x)| \leq 58$  on  $[-2, 3]$ . Find a smallest possible value of  $n$  which the error formula for Simpson's Rule ( $|E_S| \leq \frac{K^*(b-a)^5}{180n^4}$ ) guarantees can be used, where the notation used here is that generally used in class in this context. *Note: If you use a calculator prior to the very last step, you are misusing it.*
3. Without evaluating it, prove  $\int_1^{\infty} \frac{1}{x^2 + x} dx < \infty$ .
4. Evaluate  $\int_1^{\infty} \frac{1}{x^2 + x} dx$ .
5. Determine whether  $\int_0^{\infty} \frac{x^2}{e^x} dx$  is convergent. Justify your conclusion.
6. Sketch the parametric curve  $x = 5(t - \sin t)$ ,  $y = 5(1 - \cos t)$ ,  $0 \leq t \leq 4\pi$  and represent its length by a definite integral. *Extra Credit: Evaluate the integral.*
7. Sketch the polar curve  $r = 2 + \sin(4\theta)$ ,  $0 \leq \theta \leq 2\pi$  and find the area of the region it encloses.

## Extra Credit

Extra credit will be awarded for the best joke. All jokes must observe standards of good taste. The determination of the best joke will be made by popular vote in class when the papers are returned.

Please write your joke here.