Mathematics 1132 <u>SOLUTIONS</u>
Professor Alan H. Stein
Due Thursday, July 7, 2009

Let $\mathcal{C}_1$ be the portion of the graph of $y = x^2 + 5x - 1$ between the two points where the graphs of $y = 9x - 4$ and $y = x^2 + 5x - 1$ intersect. Let $\mathcal{C}_2$ be the portion of the graph of $y = 9x - 4$ between the two points where the graphs of $y = 9x - 4$ and $y = x^2 + 5x - 1$ intersect. Let $\mathcal{D}$ be the plane region bounded by $\mathcal{C}_1$ and $\mathcal{C}_2$.

For each question, sketch the relevant curve or region, which may be a plane region, solid or surface, and set up a definite integrals whose value will give the quantity indicated and, unless instructed otherwise, evaluate the integral.

Solution: First, we will obviously need to know where the curves intersect, so we solve the equations $y = 9x - 4$ and $y = x^2 + 5x - 1$ simultaneously. Using substitution, we have $x^2 + 5x - 1 = 9x - 4$, $x^2 - 4x + 3 = 0$, $(x - 1)(x - 3) = 0$, giving solutions $x = 1$ and $x = 3$. The curves therefore intersect at the points $(1, 5)$ and $(3, 23)$ and, in between, the line $y = 9x - 4$ is obviously above the parabola $y = x^2 + 5x - 1$, since the parabola is always concave up.

1. The area of $\mathcal{D}$.

   Solution: Area $= \int\limits_1^3 (9x-4)-(x^2+5x-1)\,dx = \int\limits_1^3 -x^2+4x-3\,dx = -\dfrac{x^3}{3}+2x^2-3x\Big|_1^3 = -\dfrac{3^3}{3} + 2\cdot 3^2 - 3\cdot 3 - \big(-\dfrac{1^3}{3} + 2\cdot 1^2 - 3\cdot 1\big) = -9 + 18 - 9 - (-\tfrac{1}{3} + 2 - 3) = \tfrac{4}{3}$.

2. The volume of the solid obtained by rotating $\mathcal{D}$ about the x-axis.

   Solution: Volume $= \pi \int\limits_1^3 (9x-4)^2 - (x^2+5x-1)^2\,dx =$
   $\pi \int\limits_1^3 -x^4 - 10x^3 + 58x^2 - 62x + 15\,dx = \pi\big[-\tfrac{1}{5}x^5 - \tfrac{5}{2}x^4 + \tfrac{58}{3}x^3 - 31x^2 + 15x - \tfrac{64}{27}\big]_1^3 = \tfrac{544\pi}{15}$.

3. The volume of the solid obtained by rotating $\mathcal{D}$ about the line $y = 2$.

   Solution: Volume $= \pi \int\limits_1^3 (9x-4-2)^2 - (x^2+5x-1-2)^2\,dx$
   $= \pi \int\limits_1^3 -x^4 - 10x^3 + 62x^2 - 78x + 27\,dx = \pi\big[-\tfrac{1}{5}x^5 - \tfrac{5}{2}x^4 + \tfrac{62}{3}x^3 - 39x^2 + 27x\big]_1^3 = \tfrac{464\pi}{15}$.

4. The volume of the solid obtained by rotating $\mathcal{D}$ about the y-axis.

   Solution: Volume $= 2\pi \int\limits_1^3 x[(9x-4) - (x^2+5x-1)]\,dx = 2\pi \int\limits_1^3 -x^3 + 4x^2 - 3x\,dx = 2\pi\big[-\tfrac{1}{4}x^4 + \tfrac{4}{3}x^3 - \tfrac{3}{2}x^2\big]_1^3 = \tfrac{16\pi}{3}$.

5. The length of $\mathcal{C}_1$. *Do not evaluate the integral.*

   Solution: Length $= \int_1^3 \sqrt{1 + (\dfrac{d}{dx}(x^2+5x-1))^2}\,dx = \int_1^3 \sqrt{1 + (2x+5)^2}\,dx$.

6. The area of the surface obtained by rotating $\mathcal{C}_1$ about the x-axis. *Do not evaluate the integral.*

   Solution: Surface Area $= 2\pi \int_1^3 (x^2 + 5x - 1)\sqrt{1 + (2x+5)^2}\, dx.$

7. The area of the surface obtained by rotating $\mathcal{C}_1$ about the line $y = 2$. *Do not evaluate the integral.*

   Solution: Surface Area $= 2\pi \int_1^3 (x^2 + 5x - 1 - 2)\sqrt{1 + (2x+5)^2}\, dx.$

8. The area of the surface obtained by rotating $\mathcal{C}_1$ about the y-axis. *Do not evaluate the integral.*

   Solution: Surface Area $= 2\pi \int_1^3 x\sqrt{1 + (2x+5)^2}\, dx.$