Mathematics 109   Name: _____
Professor Alan H. Stein
Due Wednesday, December 7, 2005   This problem set is worth 50 points.

Make sure that you check the course website for instructions, fill out the pledge form and hand it in with your paper. The instructions for problem sets and take-home examinations along with the pledge form are available from the *General Policies* portion of the web site. *No paper will be accepted without a signed pledge form.* Remember that your paper may be handed in before the deadline but that no late papers will be accepted regardless of the reason.

This problem set is *optional*. If it is submitted by the due date, it will be graded and used to compute your average. If it is not submitted, your average will be calculated based on your other grades. There will be no penalty for not submitting this problem set.

Note that, since most of the calculations involved can be done routinely using either a calculator or a symbolic manipulation program such as Maple or Mathematica, it will obviously be necessary to show, through your work, exactly how you came up with your solutions.

If this problem set it submitted by the next to the last class of the semester, it will be graded and returned at the last class.

Remember, wherever feasible, to give an exact value before, where appropriate, giving a decimal approximation. For example, $\sqrt{3}$ is an exact value, while 1.732050808 is just a decimal approximation; $18\sin(\pi/12)$ is an exact value, while 4.6587428 is a decimal approximation.

1. Solve the inequality $x^4 - 27x^2 + 59x \le 3x^3 + 30$.

2. John is driving east towards Waterbury at a speed of 55 miles per hour, while his wife Mary is driving south towards Waterbury at a speed of 65 miles per hour. Assuming the earth is flat and the highways they are driving on are both perfectly straight, their speeds do not vary, John is 200 miles west of Waterbury at noon while his wife is 100 miles north of Waterbury at 1 p.m., express the distance between them as a function of time. *Introduce appropriate variable with appropriate units. Make sure you clearly explain what each variable represents.*

3. Let $f(x) = x^3 - 5x + 1$. Simplify $\dfrac{f(x) - f(8)}{x - 8}$ as much as possible.

4. Solve $|x + 5| \le 12$.

5. A meteorite contains 0.038 grams of radioactive kryptonite, a substance which Superman is highly allergic to. Kryptonite has a half-life of two years and it won't be safe for Superman to touch the meteorite until it contains no more than 0.0013 grams of kryptonite. If today is Superman's eighteenth birthday, how old will be be when the meteorite is safe to touch? *Determine his age to the nearest month.*

6. A right triangle has an acute angle measuring 39° and the leg adjacent to that angle has length 49. Solve the triangle.

7. A right triangle has an acute angle measuring 39° and the hypotenuse has length 49. Solve the triangle.

Answer one of questions (8-9) without using either the Law of Sines or the Law of Cosines. Answer the other without constructing any altitudes.

8. A triangle has angles of 53° and 58° with the length of the included side being 25. Solve the triangle.

9. A triangle has sides of length 53 and 58 and the angle between them measures 25°. Solve the triangle.

10. Find $\sin(-8\pi/3)$, $\cos(-8\pi/3)$ and $\tan(-8\pi/3)$ exactly without using a calculator.

11. Find $\sin(7\pi/4)$, $\cos(7\pi/4)$ and $\tan(7\pi/4)$ exactly without using a calculator.

12. Suppose $\cos\theta = w$. Find $\cos(-\theta)$.

13. Suppose $\sin\theta = w$. Find $\sin(\pi - \theta)$.

14. Verify the identity $\dfrac{\sin\theta}{1+\cos\theta} = \csc\theta - \cot\theta$ by simplifying one of the sides until you obtain the other side.

15. Starting with the identity $\cos^2\theta + \sin^2\theta = 1$, perform the following steps in sequence, simplifying at each step as appropriate, to obtain one of the variations of this fundamental trigonometric identity.

    (a) Divide both sides by $\cos^2\theta$.

    (b) Subtract $\tan^2\theta$ from both sides.