Mathematics 109 SOLUTIONS
Professor Alan H. Stein
October 31, 2005 Graded Homework Assignment

1. Factor $x^4 - 18x^2 + 32x - 15$ completely.

   Solution: Looking at the divisors of 15: $\pm 1, \pm 3, \pm 5, \pm 15$, we find 1 is a zero of the polynomial, so $x-1$ is a factor. Dividing $(x^4-18x^2+32x-15) \div (x-1) = x^3+x^2-17x+15$, we have $x^4 - 18x^2 + 32x - 15 = (x-1)(x^3 + x^2 - 17x + 15)$.

   We now look at the divisors of 15 again and find 1 is also a zero of $x^3 + x^2 - 17x + 15$, so $x-1$ is a factor of $x^3+x^2-17x+15$. Dividing $(x^3+x^2-17x+15) \div (x-1) = x^2+2x-15$, we have $x^4 - 18x^2 + 32x - 15 = (x-1)^2(x^2 + 2x - 15)$.

   We can easily factor $x^2+2x-15 = (x+5)(x-3)$ at sight to obtain $x^4-18x^2+32x-15 = (x-1)^2(x+5)(x-3)$.

2. Solve the equation $x^4 - 18x^2 + 32x - 15 = 0$.

   Solution: From the first question, we easily see the solution set is $\{1, -5, 3\}$.

3. Solve the inequality $x^4 + 32x \geq 18x^2 + 15$.

   Solution: Subtracting $18x^2 + 15$ from both sides, we get the equivalent inequality $x^4 - 18x^2 + 32x - 15 \geq 0$, which may be written in the form

   $(x-1)^2(x+5)(x-3) \geq 0$.

   Using the zeros of the polynomial, obtained in Question 1, we divide the real line into the intervals $(-\infty, -5)$, $(-5, 1)$, $(1, 3)$, $(3, \infty)$.

   When $x > 3$, all the factors are positive, so the product is positive and x is a solution.

   When $1 < x < 3$, $(x-1)^2$ is positive, $x+5$ is positive and $x-3$ is negative, so the product is negative and x is not a solution.

   When $-5 < x < 1$, $(x-1)^2$ is positive, $x+5$ is positive and $x-3$ is negative, so the product is negative and x is not a solution.

   When $x < -5$, $(x-1)^2$ is positive, $x+5$ is negative and $x-3$ is negative, so the product is positive and x is a solution.

   When x is either -5, 1 or 3, the product is 0 and the x is a solution.

   We conclude the solution set is $\{x | x \leq -5 \text{ or } x \geq 3 \text{ or } x = 1\}$. This may also be expressed as $\{1\} \cup (-\infty, -5] \cup [3, \infty)$.

4. Devise a question that is equivalent to Question 2 but does not refer to any equation.

   Solution: Find all zeros of the polynomial $x^4 - 18x^2 + 32x - 15$.