The Natural Logarithm Function and The Exponential Function

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$\exp x = e^x$

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- $y = \ln x$ if and only if $x = e^y$
- $\ln(e^x) = x$

Other Bases

Suppose $y = b^x$. By the properties of logarithms, we can write $\ln y = \ln(b^x) = x \ln b$. It follows that $e^{\ln y} = e^{x \ln b}$. But, since $e^{\ln y} = y = b^x$, it follows that

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$$\log_b x = \frac{\ln x}{\ln b}$$

Derivatives

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If there are logs or exponentials with other bases, one may still use these formulas after rewriting the functions in terms of natural logs or the exponential function.

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Using the formula $a^x = e^{x \ln a}$, write 5^x as $e^{x \ln 5}$. We can then apply the Chain Rule, writing:

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$$y' = \frac{1}{\ln 7} \cdot \frac{d}{dx}(\ln x) = \frac{1}{\ln 7} \cdot \frac{1}{x} = \frac{1}{x \ln 7}$$