Definition of a Logarithm

(1) $y = \log_b x$ if and only if $x = b^y$

Essentially, the logarithm to the base b of a number x is the power which b must be raised to in order to obtain x.

This immediately leads to the two very useful formulas

(2)
$$b^{\log_b x} = x$$
 and $\log_b b^x = x$.
Properties of Logarithms

Each of the properties of exponential functions has an analog for logarithmic functions.

• $\log_b(xy) = \log_b x + \log_b y$

•
$$\log_b(x/y) = \log_b x - \log_b y$$

•
$$\log_b(x^r) = r \log_b x$$

• $\log_b 1 = 0.$

In other words, The logarithm of a product or quotient is the sum or difference of logarithms and the logarithm of a number to a power is the power times the logarithm of that number.