1. Sketch the feasible set for the following system of inequalities:

$$\begin{aligned} 2x - 3y &\le 7 \\ 3x + y &\ge 4 \\ x &\ge 0 \end{aligned}$$

Solution: The feasible set is an unbounded region in the right half-plane, bounded on the left by the portion of the y-axis for which $y \ge 4$, on the left and below by the portion of the line $3x + y = 4$ between the points $(0, 4)$ and $(\frac{19}{11}, -\frac{13}{11})$ and on the right and below by the portion of the line $2x - 3y = 7$ starting at the point $(\frac{19}{11}, -\frac{13}{11})$ and extending up and to the right.

2. Solve the following system of linear equations using Gaussian Elimination on the augmented matrix for the system.

$$\begin{aligned} x + z &= 4 \\ 4y - 3z &= 17 \\ 2x + 2y + z &= 9 \end{aligned}$$

Solution: The augmented matrix is $\begin{pmatrix} 1 & 0 & 1 & 4 \\ 0 & 4 & -3 & 17 \\ 2 & 2 & 1 & 9 \end{pmatrix}$.

Pivoting about the rst row, rst column: $R_3 - 2R_1$: $\begin{pmatrix} 1 & 0 & 1 & 4 \\ 0 & 4 & -3 & 17 \\ 0 & 2 & -1 & 1 \end{pmatrix}$.

Pivoting about the second row, second column:
$R_2/4$: $\begin{pmatrix} 1 & 0 & 1 & 4 \\ 0 & 1 & -\frac{3}{4} & \frac{17}{4} \\ 0 & 2 & -1 & 1 \end{pmatrix}$ $R_3 - 2R_2$: $\begin{pmatrix} 1 & 0 & 1 & 4 \\ 0 & 1 & -\frac{3}{4} & \frac{17}{4} \\ 0 & 0 & \frac{1}{2} & -\frac{15}{2} \end{pmatrix}$.

Pivoting about the third row, third column:
$2R_3$: $\begin{pmatrix} 1 & 0 & 1 & 4 \\ 0 & 1 & -\frac{3}{4} & \frac{17}{4} \\ 0 & 0 & 1 & -15 \end{pmatrix}$ $\begin{matrix} R_1 - R_3 \\ R_2 + \frac{3}{4}R_3 \end{matrix}$: $\begin{pmatrix} 1 & 0 & 1 & 19 \\ 0 & 1 & 0 & -7 \\ 0 & 0 & 1 & -15 \end{pmatrix}$

Assuming the variables are x, y and z, we thus obtain the solution $x = 19$, $y = -7$, $z = -15$.

(3-4): Let $A = \begin{pmatrix} 2 & 5 & 3 \\ 4 & 23 & 10 \\ 5 & 3 & 1 \end{pmatrix}$.

3. Perform the Elementary Row Operation $R_2 \to R_2 - 3R_1$ on A.

Solution: $\begin{pmatrix} 2 & 5 & 3 \\ -2 & 8 & 1 \\ 5 & 3 & 1 \end{pmatrix}$

4. Pivot about the 3$^{\text{rd}}$ row, 1$^{\text{st}}$ column of A.

Solution: $R_3/5$: $\begin{pmatrix} 2 & 5 & 3 \\ 4 & 23 & 10 \\ 1 & \frac{3}{5} & \frac{1}{5} \end{pmatrix}$ $\quad \begin{matrix} R_1 - 2R_3 \\ R_2 - 4R_3 \end{matrix}$: $\begin{pmatrix} 0 & \frac{19}{5} & \frac{13}{5} \\ 0 & \frac{103}{5} & \frac{46}{5} \\ 1 & \frac{3}{5} & \frac{1}{5} \end{pmatrix}$

5. Consider the following problem.

> The Nutmeg State Mining Company operates two mines for the purpose of extracting gold and sand. One costs $\$15,000$ per day to operate and yields 100 ounces of gold and 20 tons of sand each day. The other costs $\$20,000$ per day to operate and yields 120 ounces of gold and 15 tons of sand each day. The company needs to deliver at least 800 ounces of gold and 100 tons of sand. How many days should each mine be operated in order to enable the company to meet its commitments at the least cost?

Determine appropriate variables and set up appropriate constraints, non-negativity conditions and an objective function.

Solution: Let x represent the number of days the rst mine is operated and let y represent the number of days the second mine is operated. Let C represent the cost of operating the mines.

$$100x + 120y \geq 800$$
$$20x + 15y \geq 100$$
$$x \geq 0, y \geq 0$$
$$C = 15000x + 20000y$$

6. Consider a linear programming problem with the following constraints, non-negativity conditions and objective function. Assume the objective function is to be maximized. Set up the initial simplex tableau.

$$
\begin{aligned}
5x + 3y + z &\leq 90 \\
3x + 8y + 10z &\leq 55 \\
4x + 4y + 3z &\leq 63 \\
x \geq 0, y &\geq 0 \\
p = x + 5y + 2z &
\end{aligned}
$$

Solution: $\begin{pmatrix} 5 & 3 & 1 & 1 & 0 & 0 & 0 & 90 \\ 3 & 8 & 10 & 0 & 1 & 0 & 0 & 55 \\ 4 & 4 & 3 & 0 & 0 & 1 & 0 & 63 \\ -1 & -5 & -2 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$

7. Solve the following linear programming problem geometrically, where C is to be minimized.

$$
\begin{aligned}
x + 2y &\geq 40 \\
x + y &\geq 30 \\
x \geq 0, y &\geq 0 \\
C = 3x + 6y &
\end{aligned}
$$

Solution: The feasible set is the unbounded region in the rst quadrant with vertices $(0, 30)$, $(20, 10)$, $(40, 0)$. The values of the objective function at those vertices are 90, 120 and 120, respectively. Thus, the minimal value for C is 90 and it occurs when $x = 0$ and $y = 30$.

8. Use the Simplex Method to solve the linear programming problem with the following initial simplex tableau. Assume the variables are x and y, the objective function is p and is to be maximized.

$$\begin{pmatrix} 1 & 2 & 1 & 0 & 0 & 0 & 40 \\ 3 & 2 & 0 & 1 & 0 & 0 & 50 \\ 4 & 3 & 0 & 0 & 1 & 0 & 70 \\ -6 & -10 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

Solution: First pivot about the rst row, second column.

$$R_1/2: \begin{pmatrix} \frac{1}{2} & 1 & \frac{1}{2} & 0 & 0 & 0 & 20 \\ 3 & 2 & 0 & 1 & 0 & 0 & 50 \\ 4 & 3 & 0 & 0 & 1 & 0 & 70 \\ -6 & -10 & 0 & 0 & 0 & 1 & 0 \end{pmatrix} \begin{matrix} R_2 - 2R_1 \\ R_3 - 3R_1 : \\ R_4 + 10R_1 \end{matrix} \begin{pmatrix} \frac{1}{2} & 1 & \frac{1}{2} & 0 & 0 & 0 & 20 \\ 2 & 0 & -1 & 1 & 0 & 0 & 10 \\ \frac{5}{2} & 0 & -\frac{3}{2} & 0 & 1 & 0 & 10 \\ -1 & 0 & 5 & 0 & 0 & 1 & 200 \end{pmatrix}$$

Now pivot about the third row, rst column:

$$\frac{2}{5}R_3: \begin{pmatrix} \frac{1}{2} & 1 & \frac{1}{2} & 0 & 0 & 0 & 20 \\ 2 & 0 & -1 & 1 & 0 & 0 & 10 \\ 1 & 0 & -\frac{3}{5} & 0 & \frac{2}{5} & 0 & 4 \\ -1 & 0 & 5 & 0 & 0 & 1 & 200 \end{pmatrix} \begin{matrix} R_1 - \frac{1}{2}R_3 \\ R_2 - 2R_3 : \\ R_4 + R_3 \end{matrix} \begin{pmatrix} 0 & 1 & \frac{4}{5} & 0 & -\frac{1}{5} & 0 & 18 \\ 0 & 0 & \frac{1}{5} & 0 & -\frac{4}{5} & 0 & 2 \\ 1 & 0 & -\frac{3}{5} & 0 & \frac{2}{5} & 0 & 4 \\ 0 & 0 & \frac{22}{5} & 0 & \frac{2}{5} & 1 & 204 \end{pmatrix}$$

We conclude the minimum value is $p = 204$ and occurs when $x = 4$, $y = 18$.

9. Explain the di erence between a permutation and a combination.

   Solution: A combination is simply a subset while a permutation is an arrangement of elements.

10. Calculate $C(7, 2)$.

    Solution: $C(7, 2) = \dfrac{7!}{2!5!} = \dfrac{7 \cdot 6}{2 \cdot 1} = 21.$

11. Calculate $P(7, 2)$.

    Solution: $P(7, 2) = 7 \cdot 6 = 42.$

12. How many ten (10) card hands are there which contain four of one kind, three of another kind, a pair and a singleton?

    Solution: $13 \cdot 12\binom{4}{3}11\binom{4}{2}40$

(13-16): A pair of dice are rolled. Let A be the event the sum is even, let B be the event a 7 is rolled and let C be the event an 8 is rolled.

13. Find $Pr(A)$.

    Solution: $Pr(A) = \frac{1}{2}$.

14. Find $Pr(B)$.

    Solution: $Pr(B) = \dfrac{6}{36} = \dfrac{1}{6}$.

15. Find $Pr(B|A)$.

    Solution: $Pr(B|A) = 0$.

16. Find $Pr(C|A)$.

    Solution: $Pr(C|A) = \dfrac{5}{18}$.

17. What will the balance in an account be after 20 years if a single deposit of $5,000 is left alone, accumulating interest at an annual rate of 6%, compounded quarterly?

    Solution: $5000\left(1 + \frac{0.06}{4}\right)^{80}$

18. What is the present value of 20 annual payments of $5,000, assuming an annual interest rate of 6%, compounded annually? *Assume the first payment is made in one year.*

    Solution: $P = a_{\overline{n}|i}R = \dfrac{(1+.06)^{20}-1}{0.06(1+.06)^{20}} \cdot 5,000.$

19. What is the future value of 20 annual payments of $5,000, assuming an annual interest rate of 6%, compounded annually? *Assume the first payment is made in one year and you want the value as soon as the last payment is made.*

    Solution: $F = s_{\overline{n}|i}R = \dfrac{(1+.06)^{20}-1}{0.06} \cdot 5,000.$

20. What will the monthly payments be on a $100,000 loan to be repaid in 20 years if the annual interest rate is 6%?

    Solution: $P = a_{\overline{n}|i}R$, so $R = \dfrac{P}{a_{\overline{n}|i}} = \dfrac{100,000}{\dfrac{\left(1+.06/12\right)^{240}-1}{(.06/12)(1+.06/12)^{240}}}.$