Mathematics 1070 SOLUTIONS

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Note: Most calculations are approximations, even though equal signs are used. The different variables used are used in the same way the variables with the same names were used in class except where noted.

1. Consider the formula $F = P\left(1 + \frac{r}{n}\right)^{nt}$. Explain the meaning of each of the variables: F, P, r, n and t. Do so using complete sentences and without the use of any mathematical notation other than references to the variables.

   Solution: This is the formula for the future value of an asset which appreciates at a xed annual rate, compounded periodically.
   F represents the future value.
   P represents the present value.
   r represents the annual interest rate.
   n represents the number of times interest is compounded annually.
   t represents time, measured in years.

2. Consider the formula $F = s_{\overline{n}|i}R$, where $s_{\overline{n}|i} = \dfrac{(1+i)^n - 1}{i}$. Discuss the use of the formula and explain the meaning of the variables: F, n, i and R. Do so using complete sentences and without the use of any mathematical notation other than references to the variables.

   Solution: This is the formula for the future value of an increasing annuity, with $s_{\overline{n}|i}$ essentially being a multiplier representing the multiple of the periodic \rent" the future value will come out to be.
   F represents the future value.
   R represents the periodic \rent."
   n represents the number of payments.
   i represents the interest rate applied to the prior balance each time a rent payment is made.

3. Consider the formula $a_{\overline{n}|i} = \dfrac{(1+i)^n - 1}{i(1+i)^n}$. Discuss the use of the formula and explain the meaning of the variables: n and i. Do so using complete sentences and without the use of any mathematical notation other than references to the variables.

   Solution: This is used to nd the present value of a decreasing annuity, with $a_{\overline{n}|i}$ e ectively being the multiple of the periodic \rent" payment the present value is.
   n represents the number of payments.
   i represents the interest rate applied to the value of the annuity at the time each \rent" payment is made.

4. \$583.28 is placed on deposit in an account paying interest at an annual rate of 3.28%, compounded monthly. What will the balance be in ve years?

   Solution: We may use the formula $P = P_0(1 + r/n)^{nt}$ = 583.28(1 + .0328/12)$^{12\cdot 5}$ = 687.075111978, so the balance will be \$687.08.

5. \$583.28 is placed on deposit in an account paying interest at an annual rate of 3.28%, compounded continuously. What will the balance be in ve years?

   Solution: We may use the formula $P = P_0e^{rt}$ = 583.28$e^{.0328\cdot 5}$ = 687.228845706, so the balance will be \$687.23.

6. An account pays interest at an annual rate of 4.51%, compounded quarterly. What is the e ective annual yield?

   Solution: This may be done several ways. We note that \$100 will grow to 100(1 + .0451/4)$^{4\cdot 1}$ = 104.58650328 in one year, so the e ective annual yield must be 4.58650328%.

7. An account pays interest at an annual rate of 4.51%, compounded monthly. How long will it take for the balance to at least triple?

   Solution: Using the formula $P = P_0(1 + r/n)^{nt}$, we solve $3P_0 = P_0(1 + .0451/12)^{12t}$ for t:
   $(1 + .0451/12)^{12t} = 3$
   $12t \ln(1 + .0451/12) = \ln 3$
   $t = \dfrac{\ln 3}{12 \ln(1 + .0451/12)} = 24.405221164.$
   Since $405221164 \cdot 12 \approx 4.862653968$, it will take 24 years and 5 months for the balance to triple.

8. What annual interest rate is needed on an account in order for the balance to double in fteen years, assuming interest is compounded monthly?

   Solution: Again using the formula $P = P_0(1 + r/n)^{nt}$, we solve $2P_0 = P_0(1 + r/12)^{12\cdot 15}$ for r:
   $(1 + r/12)^{180} = 2$
   $(1 + r/12) = \sqrt[180]{2}$
   $r/12 = \sqrt[180]{2} - 1$
   $r = 12(\sqrt[180]{2} - 1) = .04629889908$, so we need an annual rate of 4.629889908%.