Mathematics 1070 Name: _____
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September 22, 2008

1. Define the term "set."

   Solution: A set is a collection of objects.

2. Define the term "combination."

   Solution: A combination is a subset.

3. Define the term "permutation."

   Solution: A permutation is an arrangement of elements of a set.

4. Define the term "sample space."

   Solution: A sample space is the set of possible outcomes to an experiment.

5. Define the term "probability."

   Solution: Probability is a number between 0 and 1.

6. Define the term "event."

   Solution: An event is a subset of a sample space.

7. Explain the difference between a permutation with replacement and a permutation without replacement.

   Solution: In a permutation with replacement, the same element may appear more than once; in a permuation without replacement, no element may appear more than once.

8. State the *Fundamental Theorem of Counting*, also known as *The Multiplication Principle*.

   Solution: The number of ways of making a sequence of choices is equal to the number of ways of making the first choice times the number of ways of making the second choice times the number of ways of making the third choice and so on.

9. Complete the formula $P(n,r) =$

   Solution: $P(n,r) = n(n-1)(n-2)\cdots(n-[r-1]) = n(n-1)(n-2)\cdots(n-r+1) = \frac{n!}{(n-r)!}$.

10. Complete the formula $C(n,r) =$

    Solution: $C(n,r) = \frac{n!}{r!(n-r)!}$.

11. Calculate 5!.

    Solution: $5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 120$.

12. Calculate $P(7,4)$.

    Solution: $P(7,4) = 7 \cdot 6 \cdot 5 \cdot 4 = 840$.

13. Calculate $C(7,4)$.

    Solution: $C(7,4) = \frac{7!}{4!3!} = \frac{7\cdot6\cdot5\cdot4\cdot3\cdot2\cdot1}{4\cdot3\cdot2\cdot1\cdot3\cdot2\cdot1} = \frac{7\cdot6\cdot5}{3\cdot2} = 35.$

(14-16): Let $A = \{a, b, c, d, e\}$, $B = \{a, c, e, f, g\}$.

14. Find $A \cup B$.

    Solution: $A \cup B = \{a, b, c, d, e, f, g\}$.

15. Find $A \cap B$.

    Solution: $A \cap B = \{a, c, e\}$.

16. Find $A - B$.

    Solution: $A - B = \{b, d\}$.

17. Suppose there are 25 ways of flying from Hartford to Atlanta and 15 ways of flying from Atlanta to New Orleans. How many ways are there of flying from Hartford to New Orleans with a stopover in Atlanta.

    Solution: $25 \cdot 15 = 375$.

18. A certain state has license plates which consist of two letters, followed by three digits, followed by a single character which may be either a letter or a digit. How many different license plates are possible?

    Solution: $26^2 \cdot 10^3 \cdot 36$. *Note: This comes to* $24,336,000$.

19. A major league baseball roster contains 25 players, of whom 10 are pitchers. Assuming everyone who is not a pitcher can play every other position, while the pitchers can only pitch, this is the National League so the pitcher must hit and the pitcher is going to hit ninth, how many different nine-player batting orders are possible?

    Solution: $P(15,8) \cdot 10 = 2,594,592,000$.

20. List all the possible two-letter "words" which may be formed using two different letters of the word "love." *Make sure your list is organized enough so that its completeness is obvious.*

    Solution: lo, lv, le, ol, ov, oe, vl, vo, ve, el, eo, ev

21. How many ways can a student organization with 15 members choose a president, vice president and secretary?

    Solution: $P(15,3) = 15 \cdot 14 \cdot 13 = 2,730$.

22. If a coin is tossed 5 times, how many different sequences of heads and tails is possible?

    Solution: $2^5 = 32$.

23. An urn contains 10 numbered balls, of which 6 are red and 4 are green. In how many ways can 7 balls be selected?

    Solution: $\binom{10}{7} = \frac{10 \cdot 9 \cdot 8}{3 \cdot 2 \cdot 1} = 120$.

24. An urn contains 10 numbered balls, of which 6 are red and 4 are green. In how many ways can 7 balls be selected with 4 being red and three being green?

    Solution: $\binom{6}{4} \cdot \binom{4}{3} = \frac{6 \cdot 5}{2} \cdot 4 = 60$.

25. How many different five card hands are there which consist of three of a kind and two singletons?

    Solution: $13 \cdot \binom{4}{3} \cdot \binom{12}{2} \cdot 4 \cdot 4 = 54,912$.

26. What's the probability of a five card hand containing three of a kind and two singletons?

    Solution: $\frac{13 \cdot \binom{4}{3} \cdot \binom{12}{2} \cdot 4 \cdot 4}{\binom{52}{5}} = \frac{13 \cdot 11}{4 \cdot 52 \cdot 17 \cdot 10 \cdot 49} \approx 8.98692810458 \times 10^{-5}$

27. A coin is tossed 3 times. What's the probability of getting 2 heads?

    Solution: $\frac{\binom{3}{2}}{2^3} = \frac{3}{8}$, *since there are* 2^3 *ways of tossing a coin three times and* $\binom{3}{2}$ *ways of getting two heads.*