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The variables must also satisfy the *non-negativity condition*: they can't be negative.
The set of points, or values of the variables, which satisfy the constraints and the non-negativity condition is called the *feasible set*.

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If $\alpha + \beta m > 0$, then $p$ increases when $x$ increases and $p$ decreases when $x$ decreases.

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If an extreme value occurs in a closed polygon, we easily see the extreme value must occur along the boundary, and then if it occurs along the boundary it must occur at one of the vertices, which is what the Fundamental Theorem of Linear Programming says.

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- Find the feasible set.
- Find the vertices.
- Evaluate the objective function at each vertex.
- Pick out the optimal value for the objective function.

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# The Elephant in the Room

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In particular,

- Look for variables and unknowns.
- Find connections among the variables and unknowns. In this case, the connections translate into constraints along with the objective function.

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# The Simplex Method

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In linear programming problems, we generally use the *Simplex Method*.

# An Example

Consider the following maximum problem:

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$$
\begin{aligned}
2x + y &\leq 3 \\
3x + y &\leq 4 \\
x \geq 0, y \geq 0 \\
p = 17x + 5y
\end{aligned}
$$

# Vertices

The vertices of the feasible set and the values of the objective function $p = 17x + 5y$ at those points are the following:

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The vertices of the feasible set and the values of the objective function $p = 17x + 5y$ at those points are the following:

$$\begin{array}{cc} (0,0) & 0 \\ (4/3,0) & 22\frac{2}{3} \\ (1,1) & 22 \\ (0,3) & 15 \end{array}.$$

Thus, the maximum value for $p$ is $22\frac{2}{3}$, which occurs at $(4/3, 0)$.

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Looking at the inequality $2x + y \leq 3$, we introduce the slack variable $u = 3 - (2x + y)$, so $u \geq 0$ while $2x + y + u = 3$.

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Looking at the inequality $2x + y \leq 3$, we introduce the slack variable $u = 3 - (2x + y)$, so $u \geq 0$ while $2x + y + u = 3$.

Similarly, looking at the inequality $3x + y \leq 4$, we introduce the slack variable $v = 4 - (3x + y)$, so $v \geq 0$ while $3x + y + v = 4$.

# Restating the Problem

We can thus restate the problem as

$$
\begin{aligned}
2x + y + u \quad &= 3 \\
3x + y + v \quad &= 4 \\
x, y, u, v &\geq 0 \\
p = 17x + 5y &
\end{aligned}
$$

where we find the maximum value for the objective function $p$ which satisfies the contraints, now given as equations, and the non-negativity condition.

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We will write down the augmented matrix for this system, use it to easily pick out one of the vertices and the value of the objective function at that vertex, and then pivot in a way that will enable us to find another vertex at which the value for the objective function is larger.

# The Initial Simplex Tableau

The augmented matrix for the system

$$\begin{aligned} 2x + y + u \quad &= 3 \\ 3x + y + v \quad &= 4 \\ -17x - 5y + p \quad &= 0. \end{aligned}$$

is

$$\begin{pmatrix} x & y & u & v & p & \\ 2 & 1 & 1 & 0 & 0 & 3 \\ 3 & 1 & 0 & 1 & 0 & 4 \\ -17 & -5 & 0 & 0 & 1 & 0 \end{pmatrix}$$

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*The first row isn't actually part of the augmented matrix, but we'll write it down as a reminder of which variable each column is associated with.*

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# A Solution

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We'll do this by pivoting about the column corresponding to $x$,

# The Objective Function

The last row gives a formula $-17x - 5y + p = 0$ involving the objective function. We can solve this for the objective function, getting $p = 17x + 5y$. *It may seem as if we're going around in circles, but right now we're trying to understand a process; later we will synthesize the ideas into an efficient algorithm.*

Since at the solution we found, $x = y = 0$, $p$ takes on the value 0. On the other hand, it's clear that if we found another solution where either $x$ or $y$ was positive, we'd get a larger value for $p$. It should also be clear that increasing $x$ will increase $p$ faster than increasing $y$ would, so we'll pivot in a way that will make $x$ a Group II variable.

We'll do this by pivoting about the column corresponding to $x$, in other words, we'll pivot about the first column.

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If we pivoted about the first row, we'd start by dividing the first row by 2, getting $3/2$ in the last column. We'd next subtract 3 times the first row from the second, which would give $-\frac{1}{2}$ in the last column. Since negative numbers are evil, we'll avoid that by pivoting about the second row. So we'll pivot about the entry in the second row, first column.

# The Pivot

$$\begin{pmatrix} 2 & 1 & 1 & 0 & 0 & 3 \\ 3 & 1 & 0 & 1 & 0 & 4 \\ -17 & -5 & 0 & 0 & 1 & 0 \end{pmatrix}$$

First, we divide the second row by 3 to get a 1 at the pivot point:

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To get 0's elsewhere in the first column, we'll subtract $2\times$ the pivot row from the first row and add $17\times$ the pivot row to the third row: $R_1 \leftarrow R_1 - 2R_2$, $R_3 = R_3 + 17R_2$:

$$\begin{pmatrix} 0 & 1/3 & 1 & -2/3 & 0 & 1/3 \\ 1 & 1/3 & 0 & 1/3 & 0 & 4/3 \\ 0 & 2/3 & 0 & 17/3 & 1 & 68/3 \end{pmatrix}$$

# Interpreting the New Matrix

We'll rewrite the matrix with the columns again labeled:

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The third row corresponds to the equation $\frac{2}{3}y + \frac{17}{3}v + p = \frac{68}{3}$. Once again, since $y = v = 0$, we easily get $p = \frac{68}{3}$.

This corresponds to the vertex $(4/3, 0)$, giving the value $\frac{68}{3} = 22\frac{2}{3}$ for the objective function.

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Obviously, we have found the largest possible value for the objective function.

We'll analyze what we just did, synthesize it, and come up with the *Simplex Method*.

# The Simplex Method

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3. We now write down the augmented matrix for the system of equations, with the folowing optional adjustment: Since we never pivot about the last row, the next to last column never changes. Effectively, the next to last column is unnecessary, so we don't have to write it down.

At this point, we have the *Initial Simplex Tableau*.

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Once we have chosen the pivot row and pivot column, we pivot.

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At this point, we can set the Group I variables to 0, easily find the values for the Group II variables, and the entry in the bottom right of the matrix will be the value of the objective function.

# Complications

This works well as long as all the contraints are in the form
(Linear Polynomial) $\leq$ (Positive Constant)
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Things get more complicated if the inequality goes the other way, which generally occurs when we are trying to minimize the objective function and can also occur occasionally in maximum problems. Dealing with minimum problems is another complication.

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The complication is that this is likely to lead to a negative number on the right hand side. We need to see how to deal with that.

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The problem with having a negative constant is that when we pick out what we hope will be a feasible solution, one of the variables will fail to satisfy the non-negativity condition. We deal with that via some preliminary pivoting on the Initial Simplex Tableau.

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If there is a negative entry in the last column of a tableau (excluding the bottom row), we look in that row for a negative entry.

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Note:

- If there is more than one row with a negative entry in the last column, we can look at any of those rows.
- If there is more than one negative entry in that row, we may choose any of the columns with a negative entry as the pivot column.

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When we are finished pivoting, we then merely recognize that the entry in the lower right hand corner of the final simplex tableau is the negative of minimum value for the objective function.