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That gives us a single equation in one variable, which we may solve and then substitute that solution into either original equation or, even better, into the formula we got for the other variable.

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- Raising both sides of an equation to the same power. *Beware that this step may introduce extraneous solutions.*

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3. Interchange two equations. *This is obviously legitimate but may seem pointless. It is essentially pointless if solving equations by hand but will not be pointless when instructing a computer to solve a system of equations.*

Elementary Row Operations on Matrices

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Consider the following example, where we solve a system of two equations in two unknowns, simultaneously performing analogous operations on the coefficients.

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$$\begin{aligned} 3x + y &= 11 \\ x - y &= -3 \end{aligned} \qquad \begin{pmatrix} 3 & 1 & 11 \\ 1 & -1 & -3 \end{pmatrix}$$

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Now we'll divide both sides of the first equation by 4 and simultaneously divide the coefficients in the first row of the matrix to the right by 4.

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We can read off the solution to the system from the matrix as well as from the equations.

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A matrix is generally enclosed in a large pair of parentheses.

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A system of m equations with n unknowns will yield an $m \times n + 1$ matrix, that is, a matrix with m rows and $n + 1$ columns.

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Example

Pivot about the second row, third column of the matrix
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Step 2: First subtract 7 times the second row from the first row to get: $\begin{pmatrix} -16 & -11 & 0 \\ 3 & 2 & 1 \\ 2 & 7 & 5 \end{pmatrix}$, and then subtract 5 times the second row from the third row to get: $\begin{pmatrix} -16 & -11 & 0 \\ 3 & 2 & 1 \\ -13 & -3 & 0 \end{pmatrix}$.

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- After we pivot about a given row and column, we go down one row and to the right one column and pivot about that entry if it's not 0. If that entry is 0, we first interchange that row with some row below it with a non-zero entry in that column and then pivot. If there is no non-zero entry further down in that column, we go over one row to the right and try to pivot there.
- We continue until we reach the lower right hand corner.