Solving Word Problems

The strategy for solving word problems, presented in written form, may be summarized in three words:

Read the Question!

We'll expand on how to read the question.

Strategy

- Look for variables and unknowns – represent them by symbols (letters)
- Look for connections or relationships among the variables and unknowns – these are usually expressed through equations or formulas
- Do something with the equations and formulas – solve an equation when you can; simplify what you can; manipulate what you can

Bear in mind – every piece of information should be written mathematically!

Also remember these steps do not have to be performed in order; rather, one goes back and forth between the steps, generally repeating each step several times.

Example

Consider the following word problem:

> Develop a formula for the total cost, including tax, of an item based on its price and the sales tax rate.

We begin by finding all the variables and unknowns:

- sales tax rate $= r$
- total cost $= C$
- sales price of the item $= p$
- amount of sales tax $= T$

Finding Relationships

One way to find connections and relationships is to scan the list of variables and think of whether one of them can be calculated if you know the others.

Looking through the list of variables and unknowns, it appears that one should be able to find the total cost if one knows the price and the tax. In fact, clearly, *the total cost is equal to the sales price plus the tax.* This can be directly translated into an equation, replacing total

cost by C, is equal to by an $=$ sign, the price by p, plus by a $+$ sign, and the tax by T.

We thus get:

$C = p + T$.

Another Equation or Formula

Looking through the variables, it's also clear there should be a way of calculating the sales tax if we know the tax rate and the sales price. In fact, we know the *tax will equal the tax rate times the sales price.* We can directly translate that into an equation or formula, replacing the tax by T, replacing will equal by an $=$ sign, replacing the tax rate by r, replacing times by the multiplication operator $\cdot$ and replacing the sales price by p.

We thus get:

$T = r \cdot p$.

Doing Something with the Formulas

We thus have

- $C = p + T$
- $T = rp$

It's now time to manipulate. Looking at the formulas, we hopefully notice that the second gives a formula for one of the variables in the first, so we substitute for rp for T in the first to get

$C = p + rp$.

We may notice we can simplify this to obtain

$C = (1 + r)p$, but this isn't strictly necessary.

Thoughts to Ponder

- Most of the work did not involve what most people think of as *math*.
- We never really paid attention to what was being asked for, other than noticing at the end that we had found what we wanted.
- The process, in a way, was mostly information gathering.