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- Interest Rate – used to find interest
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When you hear the word *percent*, it's a tipoff that a rate is being given rather than an absolute amount.

Compound Interest

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After three months, the account will be credited with a quarter of a year's worth of interest, or $\frac{1}{4} \cdot r \cdot P_0 = \frac{r}{4} \cdot P_0$, leaving a balance of $P_0 + \frac{r}{4} \cdot P_0 = \left(1 + \frac{r}{4}\right) P_0$.

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After another three months, the account will be credited with a quarter of a year's interest based on the new balance, so the interest credited this time will be
$\frac{1}{4} \cdot r \cdot \left(1 + \frac{r}{4}\right) P_0 = \frac{r}{4} \cdot \left(1 + \frac{r}{4}\right) P_0$, leaving a balance of
$\left(1 + \frac{r}{4}\right) P_0 + \frac{r}{4} \cdot \left(1 + \frac{r}{4}\right) P_0 = \left(1 + \frac{r}{4}\right) \cdot \left(1 + \frac{r}{4}\right) P_0 =$
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If, instead of compounding quarterly, or four times per year, interest was compounded n times per year, the formula would have n in place of 4, yielding the *Compound Interest Formula*:

$$P = \left(1+\frac{r}{n}\right)^{nt} P_0,$$

where P represents the balance after t years.

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Future Value

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If we denote future value by F, the compound interest formula implies $F = P_0\left(1 + \frac{r}{n}\right)^{nt}$, where the variables represent what they have in the past.

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- Periodic payments are made and collect interest for a number of years, essentially as a means of saving, and at the end of that period the account has grown to an amount F, the future value.

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Similarly, the third payment will grow to a future value of $R(1 + i)^{n-3}$.

Continuing, when we get to the last payment, it doesn't collect any interest at all, so it *grows* to a *future value* of just R.

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so $-iF = R - R(1+i)^n$, $-iF = R[1-(1+i)^n]$, $F = \frac{R(1-(1+i)^n)}{-i}$, or $F = \frac{(1+i)^n - 1}{i} \cdot R$.

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We sometimes write $P = a_{\overline{n}|i} \cdot R$, where $a_{\overline{n}|i} = \frac{(1+i)^n - 1}{i(1+i)^n}$.

We can use this the calculate the cost of buying an annuity that will pay back a certain monthly rent. *Of course, if we're buying it from an insurance company, the company must make a profit, so the amount the insurance company charges will be greater than the present value.*

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This may also be written as $R = \dfrac{Pi}{(1-(1+i)^{-n})}$.

Note that Pi is one month's interest on the initial balance, Note that Pi is one month's interest on the initial balance, and this gets divided by a number a little less than 1, Note that Pi is one month's interest on the initial balance, and this gets divided by a number a little less than 1, so the monthly payment will be slightly more than the amount of interest that would be paid in a month on the original amount of the loan.

Calculating the Balance on a Loan

Let P_k be the balance after k payments have been made.

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It is convenient to do this calculation using a spreadsheet.

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$P_k = (1+i)^kP - [(1+i)^{k-1} + (1+i)^{k-2} + \cdots + 1]R.$

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The coefficient of R is precisely what we found to be the value of an annuity for which the *rent* was 1 after k payments,

$$s_{\overline{k}|i} = \frac{(1+i)^k - 1}{i}.$$

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